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PRINCIPLES OF DESIGNING DISTRIBUTED NETWORKS

by

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10 October 1962

Title Page
Foreword
Abstract
Table of Contents
56 Pages of Text
Reference (2 Pages)
Distribution List

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ABSTRACT

This article describes the synthesis of distributed networks with prescribed characteristics along with the method of approximation of their characteristics. The networks consist only of cascade and parallel connections of distributed elements of equal lengths. In such networks with elements of equal lengths, similar treatment as in lumped networks would be possible with $\lambda = j \tan (\pi f / 2 f_o)$ as a variable. But here some special features are encountered with, which are very different from those in lumped networks; i.e., $\lambda = \pm 1$ will be transmission zeros, and as for network configuration, use of close coupled coils and series elements should be avoided. In spite of such restrictions on construction, an important conclusion was obtained, that a transmission function is always realizable which has transmission zeros only on the imaginary axis and at ± 1 on the complex λ- plane. The proof is presented, accompanied by the manners of construction, approximation of a characteristic with functions having poles at $\lambda = \pm 1$, especially procedures to obtain Tchebycheff characteristics and several design examples. Some citation is also made on the construction of 2-terminal distributed networks.

TABLE OF CONTENTS

1. PREFACE

It was very recent that the theory of designing networks was established for distributed networks that enables the realization of prescribed behaviors, which showed a remarkable achievement in lumped networks. There might have been ideas to substitute a lumped network by its direct corresponding distributed one; but here the first difficulty experienced is the series impedances, which, in case of parallel wires, would constitute unbalanced elements and may be subject to undesirable radiation, or, in case of a coaxial structure, require double-shielded elements. Another difficulty may appear at the connecting parts of separate sections, and some amount of phase shift would be inevitable. Therefore the direct conversion can only be a rough approximate one.

On the other hand, according to a newly developed theory, no series elements are needed, and the connecting parts are realized easily as cascade elements, leading to excellent characteristics. It was Richards[1] that gave a clue to the theory. He presented the famous "Key Theorem", which was applied to a design principle by the author,[15][16][17] followed by papers by Matsumoto and Hatori[2], Kuroda[3][8], Ozaki[4], Kasahara and Fujisawa[5][6]. It may be noteworthy that the theory has been developed mostly by our countrymen. This article is a brief record of achievement made by the writer.

2. PRELIMINARY CONSIDERATIONS

As stated in the preceding section, the networks treated in this article have following features:

(i) They are constructed only by cascade and parallel connections of elements.

In other words, there is a restriction that no series connection is permitted, in a sharp contrast with the lumped networks. Instead, cascade connections are introduced which may bring about new possibilities. Let us add two more restrictions so as to make the theory simpler.

(ii) The electrical length of all the elements are equal.

(iii) Any line that branches off from the other never joins the original.

The writer gives the name "tree-and-branch" type networks that satisfy these 3 conditions. Most of the synthesis described later is concerned with this tree-and-branch type networks. But since this status restricts the characteristics obtainable to a certain amount, some closed loops will also be permitted if necessary, under the condition below:

(iii)' Any loop consists of even number of elements.
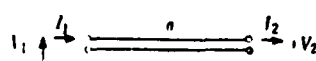
The networks will be called "normal", that satisfy the conditions (i), (ii) and (iii)'. But in this article only those closed loops are needed, that consist of 4 elements. They are termed "unit loops ".

Now, let the frequency range of interest be from 0 to f_o, and let a unit element be a line with a length equal to a quarter of the wave length at f_o. The frequency character of a normal network will be periodical with a period $(-f_o, f_o)$. Therefore f_o should be so chosen that the periodicity is of a desirable one. Here define a complex parameter

$$\lambda = j \tan \frac{\pi f}{2 f_0} \tag{1}$$

and transform the interval $(-f_o, f_o)$ into $(-j\infty, j\infty)$ on the imaginary axis. Then a distributed network can be treated in a similar way as in lumped networks.

Consider a unit element of characteristic admittance a, and rewrite the well-known transmission line equations in λ, we have (Fig. 1).

Fig. 1.   A unit element

$$V_1 = \frac{1}{\sqrt{1-\lambda^2}} V_2 + \frac{\lambda}{a\sqrt{1-\lambda^2}} I_2,$$

$$I_1 = \frac{a\lambda}{\sqrt{1-\lambda^2}} V_2 + \frac{1}{\sqrt{1-\lambda^2}} I_2 \tag{2}$$

It follows that the cascade matrix has the form

$$\frac{1}{\sqrt{1-\lambda^2}} \begin{pmatrix} 1 & \frac{\lambda}{a} \\ a\lambda & 1 \end{pmatrix}. \tag{3}$$

As is evident from Eq.(2) the input admittance of the line with the other end short- or open-circuted will be, respectively,

$$Y_{11} = \frac{a}{\lambda}, \qquad Y_{10} = a\lambda \qquad (4)$$

which are formally of the same form as in the case of an inductance or capacitance in lumped networks. Hereafter, they will be called by the same names or represented in the same notations as in lumped networks. Thus, the presence of two elements corresponding to fundamental elements in reactance networks would make it possible to construct any reactance networks without mutual inductances if series connections could be permitted. The series connections can only be admitted in the intermediate stage of synthesis, but should be completely excluded in the final networks obtained.

As already stated, the new "cascade" elements will be introduced into distributed networks. A cascade element is essentially one with 4 terminals like a gyrator or a pair of coupled coils. Since its cascade matrix is given in Eq.(3) as $\lambda \rightarrow \infty$ it reduces to

$$\begin{pmatrix} 0 & \dfrac{j}{a} \\ ja & 0 \end{pmatrix} \qquad (5)$$

which is of the same form as that of a gyrator except the fact that its entities are imaginary (of course it can never by nonreciprocal). This is the reason why a cascade element plays an important role.

Suppose that the cascade element is terminated in an admittance $Y_2(\lambda)$, the input admittance will be

$$Y_1(\lambda) = a \frac{Y_2' \lambda \cdot ja}{a + \lambda Y_2(\lambda)} \qquad (6)$$

If $Y_2(\lambda)$ is a rational function, then $Y_1(\lambda)$ will also be a rational function. It will be understood that the two-terminal admittance of a "tree-and-branch" network is always a rational function of λ. As shown later, a general normal network has a rational admittance. Hereafter it is assumed that any two-terminal admittances are rational.

As seen from Eq. (6) the degree of $Y_1(\lambda)$ will be 1 higher than that of $Y_2(\lambda)$, in general. But when $Y_2(-1) > 0$, a may be so chosen that

$$a = Y_2(-1)\qquad(7)$$

In that case the numerator and denominator of the right hand side of Eq. 6 have a common factor $1 + \lambda$, and $Y_1(\lambda)$ and $Y_2(\lambda)$ will be of the same degree after $Y_1(\lambda)$ is made into a prime form. Let us say, as seen in this case, that a network is " degenerate " if it contains cascade elements that do not effect the degree of the input admittance of the network.

Let $\lambda = 1$ in Eq. (6), one obtains

$$Y_1(1) = a\qquad(8)$$

and therefore the condition Eq. 7 may also be written $Y_1(1) = Y_2(-1)$. Furthermore, from Eq. (6) one obtains

$$Y_1(1) = Y_2(-1)$$

$$\{Y_1(1) + Y_1(-1)\}\{Y_1(1) - Y_2(-1)\} = 0\qquad(8a)$$

from which it follows that, in a nondegenerate case,

$$Y_1(1) + Y_1(-1) = 0\qquad(9)$$

Solve $Y_2(\lambda)$ from Eq. (6) one will obtain

$$Y_2(\lambda) = Y_1(1)\,\frac{Y_1(\lambda) - \lambda Y_1(1)}{Y_1(1) - \lambda Y_1(\lambda)}\qquad(10)$$

Here holds the following important theorem.

(Theorem) If $Y_1(\lambda)$ is any positive real function, $Y_2(\lambda)$ given by Eq. (10) is also positive real, and if

$$Y_1(1) + Y_1(-1) = 0\qquad(10a)$$

then the degree of $Y_2(\lambda)$ is 1 degree below that of $Y_1(\lambda)$, and if not they are of the same degree.

This is the famous "Key Theorem" due to Richards[1], and can be proved from the properties of positive real functions. The proof will be made easier by the use of the properties of echo transmission coefficients[12]. According to this theorem, any distributed admittance $Y_1(\lambda)$ can always be presented as a cascade of a unit element of characteristic admittance $Y_1(1)$ and the admittance $Y_2(\lambda)$ given by Eq. (10). Specifically, if $Y_1(\lambda)$ is a Foster, than Eq. (9) always holds and degeneracy does not occur, so that the degree reduction takes place on each separation of unit elements. Hence follows the corollary:

(Corollary) A reactance two-terminal network can be synthesized by unit elements of the same number as its degree.

This simple configuration is a specific feature of distributed networks. In a lumped network, a similar configuration, cascade of all-pass networks, may also be possible but the number of elements required will be twice the degree[20].

Here will be described certain important properties pertaining to Eq. (10). Let $Y_1(\lambda)$ be a reactance, and its zeros and poles at finite frequencies be in the order

$$0 < \omega_1 < \omega_2 < \cdots < \omega_n < \infty \tag{10b}$$

as shown in Fig. 2 (A). Say, for instance, ω_1 is a zero and ω_2 a pole, then

$$Y_2(j\omega_1) = -j\omega_1 Y_1(1), \tag{10c}$$

$$Y_2(j\omega_2) = \frac{jY_1(1)}{\omega_2} \tag{10d}$$

The former is a negative imaginary, and the latter has a positive imaginary. A zero of $Y_2(\lambda)$ should come between these two points. Similar relations hold for poles of $Y_2(\lambda)$. Thus, let the zeros and poles of $Y_2(\lambda)$ be $0 < \omega_1' < \omega_2' < \cdots < \omega_{n-1}' < \infty$, then these points will be arranged alternately in the following way:

$$0 < \omega_1 < \omega_1' < \omega_2 < \omega_2' < \cdots < \omega_{n-1}' < \omega_n < \infty \tag{10e}$$
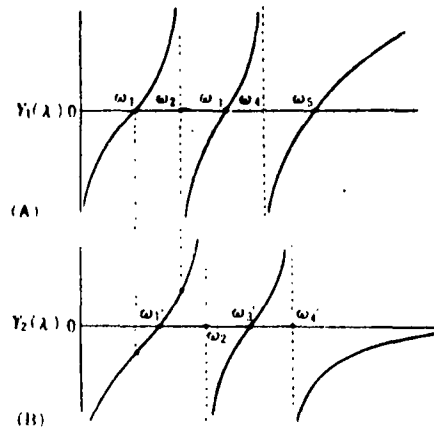
Fig. 2.   Shifting of zeros and poles of a reactance

This fact may also be interpreted as below.   Zeros (or poles) of $Y_1(\lambda)$ will, through the transformation Eq.(10) be shifted upwards, but cannot exceed the adjacent pole (or zero).   The higher of the highest of zeros and poles will go to infinity.

The properties of cascade elements have been made pretty clear.   Here the writer would like to present a rough view of parameters used in specifying 4 - terminal networks, for the convenience of the readers who are not accustomed with network theory.

A cascade matrix (hereafter called K-matrix) is usually written in the following form:

$$K = \begin{vmatrix} A & B \\ C & D \end{vmatrix}.$$

(11)

From the condition of passivity and reciprocity, there is a relation

$$AD - BC \quad 1$$

(12)

In the next place, as for the admittance matrix (Y - matrix).

$$Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$$

(13)

there is a relation $Y_{12} = Y_{21}$, and the relations between Y - matrix and K - matrix are

$$Y_{11} = \frac{D}{B}, \quad Y_{22} = \frac{A}{B}, \quad Y_{12} = -\frac{1}{B} \tag{14}$$

Impedance matrices are not used in this article, but the open-circut admittances Y_{10}, Y_{20} may be used in their places:

$$Y_{10} = \frac{1}{Z_{11}} = \frac{C}{A}, \quad Y_{20} = \frac{1}{Z_{22}} = \frac{C}{D} \tag{15}$$

The input admittance for unit admittance termination is

$$Y = \frac{C+D}{A+B} = Y_{11} - \frac{Y_{12}^2}{Y_{22}+1} = Y_{11}\frac{Y_{20}+1}{Y_{22}+1} \tag{16}$$

Normalization of terminating admittance to unity does not lead to loss of generality. Therefore all terminating admittances are considered to be 1 except for the case of matching networks.

To specify a 4 - terminal network, its K-matrix or Y-matrix will suffice. Sometimes Y_{11} and Y_{10} (or Y_{22} and Y_{20}) will be given, or only the input impedance will be given. In the former cases, the only undertermined are the ideal transformers at the output ends, whereas in the latter cases, an all-pass in the output terminal will give no change, but the amplitude-transmission characters are completely determined, and it will lead to a sufficient answer to the problems of filter design, etc.

3. PROPERTIES OF DISTRIBUTED 4 - TERMINAL NETWORKS.

If a unit element is considered to be a 4 - terminal network, its K-matrix will have a form Eq. (3) The K-matrix of a network consisting of n such elements (called a "bar" network) should be
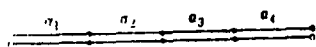

$$K = \frac{1}{(\sqrt{1-\lambda^2})^n} \begin{bmatrix} v_1(\lambda) & u_1(\lambda) \\ u_2(\lambda) & v_2(\lambda) \end{bmatrix} \tag{17}$$

Fig. 3. A bar network

since it is a product of n matrices of the form Eq.(3). Here $v_1(\lambda)$, $v_2(\lambda)$ are even polynomials with constant terms 1, $u_1(\lambda)$, $u_2(\lambda)$ are odd polynomials. If n is odd, then v_1 and v_2 are of degree n, u_1 and u_2 are of degree n - 1; if n is even, vice versa. The degree of the K-matrix is defined to be the highest of the degrees of the four polynomials. Among these four polynomials there is a relation

$$v_1(\lambda)\, v_2(\lambda) - u_1(\lambda)\, u_2(\lambda) = (1-\lambda^2)^{n-1}$$

(18)

which is derived from Eq.(12). Entries of Y-matrix may be represented by the polynomials

$$Y_{11}(\lambda) = \frac{v_2(\lambda)}{u_1(\lambda)}, \qquad Y_{22}(\lambda) = \frac{v_1(\lambda)}{u_1(\lambda)},$$

$$Y_{12}(\lambda) = -\frac{(\sqrt{1-\lambda^2})^n}{u_1(\lambda)}.$$

(19)

It is a specific feature of a distributed network that $Y_{12}(\lambda)$ has a factor $(\sqrt{1-\lambda^2})^n$. If the entries of a Y-matrix have the form Eq.(19) and if $v_1(\lambda)$, $v_2(\lambda)$ are polynominals with constant terms 1, and if $Y_{11}(\lambda)$, $Y_{22}(\lambda)$ are reactance functions of degree n, and if a function $u_1(\lambda)$ exists that satisfy Eq.(18) then the network can be realized as a bar network. The last condition above cited may be expressed that all determinants of residues of poles of Y-matrix should be zero (residues of poles should be perfectly compact, by Ozaki's words).

One more thing should be added in connection with a bar network. Of the 4 polynominals, if one odd and one even polynomials are known, the remaining two polynomials are uniquely determined. That is, if one of Y_{11}, Y_{22}, Y_{01} or Y_{02} is known, the network will be perfectly determined. The reason is simple, let, say, Y_{22} is given, then since it is a reactance of degree n, it can be realized by a cascade connection of n-elements. Put two terminals at the very end, and the network is obtained, because there can be no other networks that have the same $Y_{22}(\lambda)$.

Let us go a step further, into a network whose junctions have open-circuit elements in shunt, as shown in Fig. 4. Such a network is called a simple

Fig. 4. A simple open-branch network

open-branch network. The K-matrix of a shunt element with characteristic admittance b will be

$$\begin{bmatrix} 1 & 0 \\ \lambda b & 1 \end{bmatrix}. \tag{20}$$

Therefore the K-matrix of a simple open-branch network will be represented as a product of K's of the form Eq. (3) and K's of the form Eq. (20). Its form is the same as Eq. (17), where the constant terms of $v_1(\lambda)$ and $v_2(\lambda)$ are 1, relation Eq. 18 also holds, and the entries of K-matrix have the form Eq. (19). The only difference is that in the present case, the degree of K-matrix is higher than n. Let the number of shunt elements be ν, then the total number of elements is $n + \nu$, and the degree of K-matrix can be $n + \nu$ at most. This may not always be equal to $n + \nu$. For instance, in case Y_{11} and Y_{10} have poles at $\lambda = \infty$, the addition of shunt element λb at the input terminal will have no effect on the degrees of Y_{11} and Y_{10}. In these cases, there exist equivalent networks with a number of elements equal to the degree.

A more general 4-terminal network shall be considered. Take the case shown in Fig. 5, where the shunt elements are replaced by more general
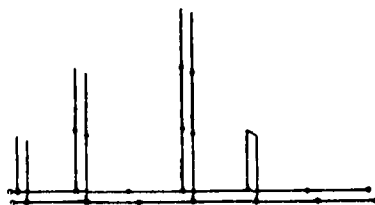

Fig. 5. A tree-and-branch network

reactances. λb in Eq. (20) should be replaced by the corresponding reactances, and K-matrix will take, in general, a form

$$\frac{1}{(\sqrt{1-\lambda^2})^l f(\lambda)} \begin{vmatrix} v_1(\lambda) & u_1(\lambda) \\ \dfrac{w_2(\lambda)}{\lambda} & v_2(\lambda) \end{vmatrix} \tag{21}$$

where $v_1(\lambda)$, $v_2(\lambda)$, $w_2(\lambda)$ and $f(\lambda)$ are even polynomials, and $u_1(\lambda)$ is an odd one. Among the constant terms of $v_1(\lambda)$, $v_2(\lambda)$ and $f(\lambda)$, there are the following relations

$$v_1(0) \geq f(0) > 0, \quad v_2(0) \geq f(0) > 0. \tag{22}$$

Therefore, amond 4 entries A, B, C and D of K-matrix, only C can have a pole at $\lambda = 0$, and it can be simple at most. If all the other ends of the shunt lines are open-circuited, even this pole vanishes and K-matrix take the following form

$$\frac{1}{(\sqrt{1-\lambda^2})^l f(\lambda)} \begin{vmatrix} v_1(\lambda) & u_1(\lambda) \\ u_2(\lambda) & v_2(\lambda) \end{vmatrix}. \tag{23}$$

It should be noted here that the constant terms of $v_1(\lambda)$, $v_2(\lambda)$ and $f(\lambda)$ become equal. Call such a network an "open-branch" network.

It is needless to say that the relation

$$v_1(\lambda) v_2(\lambda) - u_1(\lambda) \frac{w_2(\lambda)}{\lambda} = (1-\lambda^2)^l f(\lambda)^2, \tag{24}$$

holds for Eq. (21), and the relation

$$v_1(\lambda) v_2(\lambda) - u_1(\lambda) u_2(\lambda) \cdot (1 - \lambda^2)^l f(\lambda)^2 \tag{25}$$

for Eq. (23).

Y-matrices will be found to have the form

$$Y_{11}(\lambda) = \frac{v_2(\lambda)}{u_1(\lambda)}, \quad Y_{22}(\lambda) = \frac{v_1(\lambda)}{u_1(\lambda)},$$

$$Y_{12}(\lambda) = -\frac{(\sqrt{1-\lambda^2})^l f(\lambda)}{u_1(\lambda)} \tag{26}$$
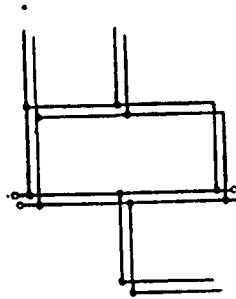
Fig. 6.   A network with a loop

Roots (zeros) of $f(\lambda)$ are poles of shunt admittances and lie only on the imaginary axis, as they should be.

Next, the case will be considered, which has a closed loop or loops. A simple example is shown in Fig. (6), which consists of two tree-and-branch networks in parallel. Its Y-matrix is the sum of those of constituent networks; and so $Y_{11}(\lambda)$ and $Y_{22}(\lambda)$ are evidently rational reactances, and $Y_{12}(\lambda)$ has the following form

$$Y_{12}(\lambda) = -\frac{(\sqrt{1-\lambda^2})' f(\lambda)}{u_1(\lambda)} - \frac{(\sqrt{1-\lambda^2})'' f(\lambda)'}{u_1(\lambda)'}$$

$$= -\frac{(\sqrt{1-\lambda^2})' \{f(\lambda) u_1(\lambda)' + (\sqrt{1-\lambda^2})''^{-'} f(\lambda)' u_1(\lambda)\}}{u_1(\lambda) u_1(\lambda)'} \tag{27}$$

where it is taken that $l'>l$.

The number of elements building up a closed loop is even, so that the sum of l and l', numbers of cascade elements in both constituent networks, is even, and their difference is also even. Therefore the expression in $\{\ \}$ in Eq. (27) is a rational polynomial. It follows that the entries of Y-matrix for this case has the same form as Eq. (26). A similar consideration goes also for more complicated cases. But, in general, as to be seen from Eq. (27) the zeros of the polynomial in $\{\ \}$ are not constrained on the imaginary axis.

From the above consideration, entries of Y-matrix of a normal reactance 4-terminal network should have the following properties:

(i) They have the form Eq.(26),

(ii) $Y_{11}(\lambda)$ and $Y_{22}(\lambda)$ are rational reactance functions,

(iii) Determinants of residues of poles are non-negative,

(iv) $v_1(o) \geq f(o) > o$, $v_2(o) \geq f(o) > 0$

Of these conditions, (ii) and (iii) are such that should be for reactance 4-terminal network. These are the conditions necessary for a normal network, but not sufficient. If the series elements and ideal transformers are permitted, these conditions become sufficient.

Conditions on K-matrix may also be stated. The K-matrix of a normal network should have the following properties:

(i) They have the form Eq.(21),

(ii) $Y(\lambda) = \dfrac{v_2(\lambda) + \dfrac{w_2(\lambda)}{\lambda}}{v_1(\lambda) + u_1(\lambda)}$ is a positive real function,

(iii) Eq.(24) holds

(iv) $v_1(o) \geq f(o) > o$, $v_2(o) \geq f(o) > o$

Further, let us call the case an "open-branch" network, where the equalities hold in (iv).

Now, several theorems will be described that will be of use in the transformation of networks. The first one corresponds to Richards' Theorem extended into open-branch 4-terminal networks.

(Theorem) If the Y-matrix of a 4-terminal network satisfies the 4 conditions above described, with equality in (iv) and $l \geq 1$, then, after separating a unit element $Y_{11}(1)$ from the input side as a cascade element, the entries of the Y-matrix of the remaining network can be represented by

$$Y_{11}'(\lambda) = Y_{11}(1) \frac{Y_{11}(\lambda) - \lambda Y_{11}(1)}{Y_{11}(1) - \lambda Y_{11}(\lambda)},$$

$$Y_{22}'(\lambda) = \frac{Y_{11}(1) Y_{22}(\lambda) - \lambda d}{Y_{11}(1) - \lambda Y_{11}(\lambda)},$$

$$Y_{12}'(\lambda) = \frac{\sqrt{1 - \lambda^2}\, Y_{11}(1) Y_{12}(\lambda)}{Y_{11}(1) - \lambda Y_{11}(\lambda)}.$$

$$(28)$$

where
$$\Delta = Y_{11}(\lambda) Y_{22}(\lambda) - Y_{12}(\lambda)^2 \tag{28a}$$

and these again satisfy the above 4 conditions with equality in (iv).

This theorem can easily be proved from the properties of the Y-matrix and those of positive real functions. This theorem is also true in certain cases of networks that are not open-branch ones.

This theorem states, with regard to the K-matrix, that the matrix, obtained by multiplying the matrix Eq. (23) by the reciprocal

$$\frac{1}{\sqrt{1-\lambda^2}} \begin{vmatrix} 1 & -\dfrac{\lambda}{a} \\ -\lambda a & 1 \end{vmatrix} \tag{29}$$

of the matrix Eq. (3) from the left, will also satisfy the conditions of the K-matrix of an open-branch network. Here, in the above expression,

$$a = Y_{11}(1) = \frac{v_2(1)}{u_1(1)} = \frac{u_2(1)}{v_1(1)} = Y(1) \tag{30}$$

The following theorem is an important one in converting a series element into a shunt one. (Fig. 7).

(A) (B)
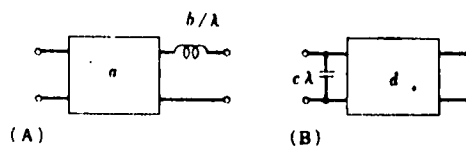
Fig. 7. Transformation between a series element and a shunt element

(Theorem) A network, built up with a unit element a with a series inductance b/λ at one end, is equivalent to another network built up with a unit element d with a shunt capacitance cλ at the other end. Here the values of the elements should satisfy the following relations.

or

$$c = \frac{a^2}{a+b}, \quad d = \frac{ab}{a+b}$$

$$a = c+d, \quad b = \frac{d^2}{c} + d. \tag{31}$$

Kuroda gave rules of transformation for the cases of a series capacitance, a shunt inductance as well as of a resonant circuit, but all these cases either ideal transformers or close coupled coils are necessary. These transformations are, in general, a kind of inverting the order of transmission zeros in cascade networks, and one of the papers treating this problem in lumped domain was prepared by Yamamoto[21].

The following corollary comes out directly from the above theorem.

(Corollary) The network Fig. 8 (a) has always an equivalent network Fig. 8 (b),

(A)   (B)
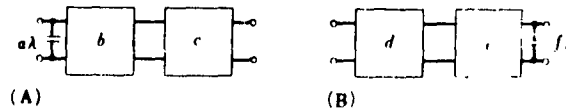
Fig. 8.   Transposition of a shunt capacitance

wherein the values of the elements have the following relations

$$d = a+b, \quad e+f = c,$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{d} + \frac{1}{e}. \tag{32}$$

A theorem will be presented which is of use in eliminating negative elements.

(Theorem) Take a Bruner section with a negative inductance in series with a unit element as shown in Fig. 9 (a),

A)   (B)

If
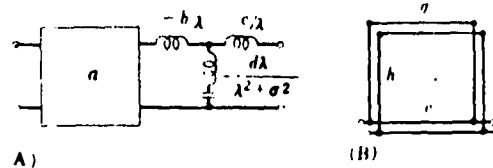
Fig. 9.   Transformation into a unit loop

$$b \cdot c+d \tag{32a}$$

$$a \cdot b \tag{32b}$$

then it can be transformed into an equivalent unit loop with no negative elements (all alphabets are to be of positive values). The conversion equations of the elements are given as follows

$$c = \frac{abc'(1+\sigma^2)}{p}$$

$$f = \frac{cd(b-a)}{p},$$

(33)

$$g = \frac{bd(b+a\sigma^2)}{\sigma^2 p},$$

where

$$h = \frac{ad(b+a\sigma^2)}{p}.$$

(33a)

$$p = b^2 + \sigma^2(ad+bc).$$

The equivalence may be readily seen if one calculates the Y-matrices of both networks. The network Fig. 9 (a) can also be transformed into the form Fig. 10,
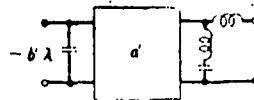
Fig. 10. Equivalent network of Fig. 9(a)

where the condition of capability of transformation without negative elements, that is, the condition which corresponds to a $\geq$ b in the above, is that only the capacitance at the left end is negative and all other elements have positive values. The condition of close coupling takes the form

$$\frac{a'^2}{b'} = a'+c+d$$

(33b)

Moreover, if there follows another cascade element to the right, the inductance c/λ can also be transformed into a capacitance. Consequently a very important result can be obtained that any standard network, in which capacitances and resonant circuits are connected in turn alternately separated by cascade elements, can always be transformed into a network with only positive elements, if it is a passive network, even if some the shunt capacitances are negative.

Lastly, a short description will also be made on the case where the inductance of the resonant arm is negative, as shown in Fig. 11. This is the
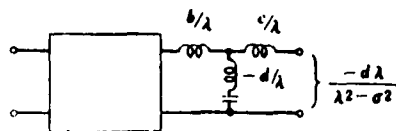
Fig. 11. The case of real zero

case where the resonance frequency, and consequently the zero of $Y_{12}(\lambda)$ is on the real λ-axis. Comparing this case with the former, the only difference is the substitution

$$-b \rightarrow b, \quad d \rightarrow -d, \quad \sigma^2 \rightarrow -\sigma^2 \tag{33c}$$

and the transformation representations into a unit loop can readily be obtained from those for the former case. As one will see, the condition of getting no negative elements is

$$\sigma^2 \geq 1$$

$$\tag{34}$$

4. CONSTRUCTION OF 4-TERMINAL NETWORKS

In this chapter it will be discussed how to construct a network with prescribed 4-terminal parameters. The prescriptions may be made in K-matrix, Y-matrix or input impedance; but K-matrix will be the most convenient in considering conditions of realizability, because in Y-matrix or input impedance some common factors in the numerator and denominator may be cancelled.

Now, a K-matrix has in general the form Eq.(21), and the Y-matrix thereof is given by Eq.(26). It has already been described that there are the relations

$$v_1(0) \geq f(0) > 0, \quad v_2(0) \geq f(0) > 0 \tag{35}$$

When inequalities hold in these expressions, residues at $\lambda = o$ are not compact. Then surplus residues can be taken out as shunt inductances as shown in Fig. 12, so that the 3 residues of the remaining network are equal. That is, let

Fig. 12. Transformation into an open-branch network

$$\left[\frac{u_1(\lambda)}{\lambda}\right]_{\lambda=0} = \alpha \qquad (36)$$

then the residues of Y_{11}, Y_{22} and Y_{12} will be

$$\frac{v_2(0)}{\alpha}, \quad \frac{v_1(0)}{\alpha}, \quad \frac{f(0)}{\alpha}, \qquad (36a)$$

so that the inductances to be taken out from the input and output ends are

$$\frac{v_2(0)-f(0)}{\alpha\lambda}, \quad \frac{v_1(0)-f(0)}{\alpha\lambda} \qquad (37)$$

Needles to say, these can be realized by elements of characteristic admittances $\{V_2(o) - f(o)\}/\alpha$ and $\{V_1(o)-f(o)\}/\alpha$ with far ends short-circuited. As for the remaining network, equalities hold in Eq.(35), so that the constant term of $W_2(\lambda)$ should be 0, which means that the K-matrix should take the form Eq.(23) and the network must be of an "open-branch" type. Consequently the whole problem will be solved if the synthesis of open-branch networks is established. For that reason, the discussion will be constrained only to open-branch networks. Of course, it is also possible to take out shunt inductances in the intermediate parts, not at the beginning, and get sometimes element values easier for realization. The way of taking out shunt inductance at first is only for the purpose of making the treatment simplier.

Now, let the prescribed K-matrix be

$$K = \frac{1}{(\sqrt{1-\lambda^2})^i f(\lambda)} \begin{bmatrix} v_1(\lambda) & u_1(\lambda) \\ u_2(\lambda) & v_2(\lambda) \end{bmatrix} \qquad (38)$$

Here the relation

$$v_1(\lambda) v_2(\lambda) - u_1(\lambda) u_2(\lambda) = (1-\lambda^2)^i f(\lambda)^2 \qquad (39)$$

holds. As is clear from the expressions of operating transmission coefficients to be described in the next chapter, roots of $f(\lambda)$ give transmission zeros of the network, and most of them lie on the imaginary axis of λ in practical filters. For this reason, only the case will be treated where the roots of

$f(\lambda)$ lie only on the imaginary axis. This is, as stated in the preceeding section, a necessary condition to be a tree-and-branch type network, and the procedure described below are, in general, to get to such structures.

Multiply the 5 polynomials of K-matrix by an appropriate constant, so as to make the constant terms of the even polynomials unity. Then $f(\lambda)$ may be written in the form:

$$f(\lambda) = \left(1+\frac{\lambda^2}{\sigma_1^2}\right)\left(1+\frac{\lambda^2}{\sigma_2^2}\right)\cdots\cdots\left(1+\frac{\lambda^2}{\sigma_m^2}\right)$$

$$0 < \sigma_1 \leq \sigma_2 \leq \cdots\cdots \leq \sigma_m < \infty$$

(40)

Here, the number of transmission zeros at finite frequencies is taken to be m; another transmission zero at $\lambda = \infty$ may also be possible. Let the degree of the K-matrix be n, then the multiplicity r of the transmission zeros at $\lambda = \infty$ be

$$r = n - 2m - l$$

(41)

Considering that the transmission zeros at finite frequencies make pairs (positive and negative), and also that there are $l/2$ transmission zeros at $\lambda = \pm 1$, the above expression shows that the total number of transmission zeros is equal to the degree of the K-matrix, as should be.

The general network realizing such a K-matrix will take a form shown in Fig. 13 (a). This may be rewitten in the same form Fig. 13 (b), which

(A)   (B)
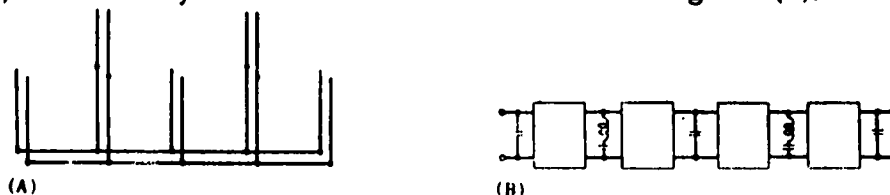
Fig. 13. Standard form of a tree-and-branch network

corresponds to a ladder type structure Fig. 14 in lumped networks. The
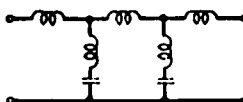
Fig. 14. A ladder network

series inductances are changed into shunt capacitances between cascade elements. Consequently the synthesis procedures are also similar. First obtain the input admittance

$$Y(\lambda) = \frac{v_2(\lambda) + u_2(\lambda)}{v_1(\lambda) + u_1(\lambda)} \tag{42}$$

from the prescribed K-matrix. As was the case in ladder networks, it is necessary to have a simple transmission zero at $\lambda = \infty$ in order to have no negative elements. Thus, one has

$$Y(\infty) = \infty \quad \text{or} \quad Y(\infty) = 0 \tag{42a}$$

Even in the latter case, the former relation will hold if a cascade element would be separated. Therefore only the former case will be considered. Get

$$C_{1n} = \left[\frac{Y(\lambda)}{\lambda} \right]_{\lambda = \infty} \tag{43}$$

This is the maximum capacitance that can be taken out at the first stage. The value of $Y(\lambda)$ at a finite transmission zero will be, by Eq. (39)

$$Y(j\sigma_\nu) = \frac{v_2(j\sigma_\nu)}{u_1(j\sigma_\nu)} = \frac{u_2(j\sigma_\nu)}{v_1(j\sigma_\nu)} = \text{imaginary} \tag{43a}$$

Therefore

$$C_{1\nu} = \frac{Y(1) - j\sigma_\nu Y(j\sigma_\nu)}{1 + \sigma_\nu^2}, \tag{44}$$

becomes real. Obtain the values of $C_{1\nu}$ for all transmission zeros; some of them may be negative, and some positive. Let the smallest among C_{10} and those of positive values of $C_{1\nu}$ be C_1, and take out λC_1 as a shunt element. The remaining admittance is

$$Y_1(\lambda) = Y(\lambda) - \lambda C_1, \tag{45}$$

Let $C_1 = C_{1\mu}$, then by Eq. (44),

$$Y_1(1) = j\sigma_\mu Y_1(j\sigma_\mu), \tag{46}$$

Since $C_{1\mu} < C_{10}$, it follows that $Y_1(\infty) = \infty$. Take out a cascade element $Y_1(1)$ from $Y_1(\lambda)$, then the remaining admittance will be

$$Y_2(\lambda) = Y_1(1) \frac{Y_1(\lambda) - \lambda Y_1(1)}{Y_1(1) - \lambda Y_1(\lambda)}, \tag{47}$$

This is seen to have a pole at $\lambda = \sigma_\mu$ by Eq. (46). Take out this pole as a resonant arm, That is

$$Y_3(\lambda) = Y_2(\lambda) - \frac{a_\mu \lambda}{\lambda^2 + \sigma_\mu^2}. \tag{48}$$

where

$$a_\mu = \left[Y_2(\lambda) \frac{\lambda^2 + \sigma_\mu^2}{\lambda} \right]_{\lambda = j\sigma_\mu} \tag{48a}$$

This resonant arm can be represented by the cascade of two elements with values $a_\mu / (1 + \sigma_\mu^2)$ and $a_\mu / \{(1+\sigma_\mu^2)\sigma_\mu^2\}$. Here also

$$Y_2(\infty) = Y_3(\infty) = 0 \tag{48b}$$

Therefore after taking out a cascade element $Y_3(1)$, the remainder $Y_4(\lambda)$ satisfies $Y_4(\infty) = \infty$. Thus a stage has been completed which corresponds to a transmission zero $j\sigma_\mu$. The procedure will be repeated, just as in the way followed in the original $Y(\lambda)$.

If, in the first step, C_{10} is the smallest, the pole of $Y(\lambda)$ at $\lambda = \infty$ is completely taken away, resulting in a transmission zero. If any more transmission zeros remain at $\lambda = \infty$, one will have $Y_1(\infty) = 0$, and after taking out a cascade element $Y_1(1)$ the remainder will become $Y_2(\infty) = \infty$. The first stage ends here in this case. In ordinary cases transmission zeros at $\lambda = \infty$ will be taken out after all the transmission zeros at finite frequencies are taken out. After all the transmission zeros (including $\lambda = \infty$) are taken out,

$Y(\lambda)$ may not still be equal to a constant 1, and then cascade elements will be taken off one after another, and finally one will get $Y(\lambda)=1$, which is the terminating admittance, and the whole synthesis ends. This is because the K-matrix with no transmission zeros on the imaginary axis should have the form Eq. (17) and should satisfy the condition of a "bar" network.

In the above description, smallest $C_{1\nu}$ has been chosed in the first stage, but in some cases those not smallest may also do as well. The above procedure may be applied to $Y_{11}(\lambda)$ in a similar manner, but a special care will be needed if there are common factors in the numerator and the denominator of $Y_{11}(\lambda) = v_2(\lambda)/u_1(\lambda)$. That is, this common factor cancelled is a transmission zero, as to be seen from Eq. (39), and even if $Y_{11}(\lambda)$ is made ∞ at this point, it is not sure that the whole residue of the pole could be taken out. In such cases one should consider $Y_{10}(\lambda)$ along with $Y_{11}(\lambda)$. Moreover, following the procedure and at the last stage one will obtain $Y_{22}(\lambda)$; if it is smaller than the prescribed $Y_{22}(\lambda)$, one must connect needy admittances in shunt to the output terminals.

Kuroda and others use the following procedure instead of the above stated one. First take out a cascade element $Y(1)$ from $Y(\lambda)$. Then take out an appropriate series inductance, and then take out a resonant arm. After that, take out two cascade elements and again take out a series inductance, and so on. Apply transformation described in the preceding section and change series inductances into shunt capacitances. Or, take out transmission zeros as a ladder network, and synthesize the remainder as a "bar" network, and the whole network is transformed into a "tree-and-branch" one by the use of transformation formulae given by Kuroda. The computation will, however, be rather complicated.

The synthesis procedure has been described so far, but it is not always possible to complete the synthesis with only the conditions stated in the preceding section. It is very difficult to obtain necessary and sufficient conditions thereof, so that the following is concerned only to the sufficient conditions.

In performing the above procedure, two cascade elements are needed to take out a transmission zero at a finite frequency, and when all finite zeros are taken out, a pole at $\lambda = \infty$ is necessarily taken out along with. Therefore the number 1 of the cascade elements must satisfy

$$l \geq 2m + r - 1 \tag{49}$$

where r is the multiplicity of transmission zeros at $\lambda = \infty$. Of course those cases should be excluded where two zeros are taken out at a time.
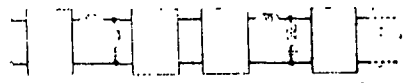
Fig. 15. Alternate way of synthesis

Next comes the problem of negative elements. In lumped networks they may be realizable if close coupled coils are permitted. In distributed networks there are no corresponding ones, and would not be realizable in the original forms. The use of unit loops described in a previous section will bring out a certain extent of possibility, but here the discussion will be constrained only to tree-and-branch networks. Kasahara and Fujisawa have given a sufficient condition to have no negative elements. It is stated as follows.

"$\lambda = \infty$ should be a transmission zero, and any of the finite real frequency transmission zeros $\sigma_1, \sigma_2, \ldots, \sigma_m$ should not be smaller than any of the positive real frequency roots of the four polynomials $v_1(\lambda)$, $v_2(\lambda)$, $u_1(\lambda)$ and $u_2(\lambda)$. "

However this condition can be extended a bit, as can be seen from the proof below. Consider two admittances

$$Y_{11}(\lambda) = \frac{v_2(\lambda)}{u_1(\lambda)}, \quad Y_{10}(\lambda) = \frac{u_2(\lambda)}{v_1(\lambda)} \tag{50}$$

Here even if the numerators and denominators have common factors, they should not be cancelled, but it should, in spite, be considered that the poles

and zeros are close to each other. Now, if $Y(\infty)=\infty$, then $Y_{11}(\infty)=\infty$ and

$Y_{10}(\infty)=\infty$, and if $\lambda=\infty$ is a transmission zero of multiplicity equal or higher

than 2, the residues of the three admittances are equal, and if the multiplicity

is 1, the residue of either Y_{11} or Y_{10}, that has higher degree, will be equal

to the residue of $Y(\lambda)$. Furthermore, the residue of the admittance of the

higher degree is smaller than that of the lower degree. This can easily be

seen from

$$v_1(\lambda)\,u_1(\lambda)\left\{\frac{v_2(\lambda)}{u_1(\lambda)} - \frac{u_2(\lambda)}{v_1(\lambda)}\right\} = (1-\lambda^2)^i\,f(\lambda)^i \tag{51}$$

which is deduced from Eq. (39), but may also be considered to be true from

a physical sense. Moreover, the highest zero of the admittance of the

higher degree is greater than that of lower degree. Therefore the above

stated condition is that any finite real frequency transmission zero should not

be smaller than the highest zero of either $Y_{11}(\lambda)$ or $Y_{12}(\lambda)$, which has the

higher degree. Let that of the higher degree be denoted by $Y_1(\lambda)$, then its

degree is n, equal to that of the K-matrix. Arrange its zeros and poles and

give them ordinal numbers beginning from small to large, as shown in Fig.16a.
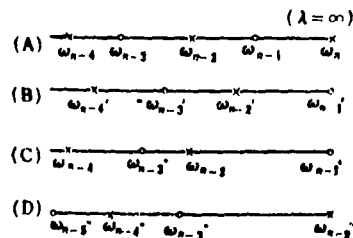
Fig. 16. Shifting of zeros and poles

After taking out a cascade element from $Y_1(\lambda)$, the zero ω_{n-1} will move to

∞, the pole ω_{n-2} to ω_{n-2}' which is smaller than ω_{n-1}, as shown in b.

Or, if the pole at $\lambda=\infty$ [equal to that of $Y(\lambda)$, and can be taken out] of $Y_1(\lambda)$

is taken out as a shunt capacitance, then ω_{n-1} again moves to ∞, and other

zeros move a bit upwards, whereas the poles do not move, as shown in c.

Therefore, after taking out a cascade element, a pole ω_{n-2} moves to ∞,

ω_{n-3} to a point below ω_{n-2}, as can be understood from these considerations,

a pole ω_{n-2} can be shifted to any point between ω_{n-2}' and ∞, if the capacitance

to be taken out first is properly chosen. Since ω_{n-2}' is smaller than ω_{n-1}', the pole can be moved to a transmission zero, if the transmission zero is above the highest zero ω_{n-1}. In this case, $Y(\lambda)$ will naturally have a pair of poles, and since the residues are generally equal, the pair of poles can be completely taken off as a resonant arm. If $Y_1(\lambda)$ had a common factor, corresponding to this pole, in its numerator and denominator, some part of residue will remain unseparated; but this status can be considered that 2 poles are duplicated, so that one may take the matter to have one of them separated. Upon separating the pole, the zero ω_{n-3} goes to ∞. In the next, upon separating a cascade element, the pole ω_{n-4} goes to ∞. Then the highest zero will be ω_{n-5}. In any step of operation, a zero or pole can not go beyond the adjacent pole or zero, so that ω_{n-5} will be below ω_{n-2}. Thus again all transmission zeros are located above the highest zero (ω_{n-5}), leading to the possibility of further synthesis.

Further, the highest zero in the second step is below the highest pole at the beginning, so that it will be comprehended that the following condition is also sufficient.

"$\lambda = \infty$ should be a transmission zero, and all other transmission zeros should not be smaller than the greatest root of $v_1(\lambda)$ or $u_1(\lambda)$, and at least one of them should not be smaller than the greatest root of $v_2(\lambda)$ or $u_2(\lambda)$. "

Through above discussions, conditions of tree-and-branch networks have been revealed to some extent. Now the restrictions on the structure will be a little loosened, permitting the use of unit loops. This, in effect, to permit the presence of negative shunt capacitances in tree-and-branch networks, and such a network may also be called "a tree-and-branch network in a broader sense". A sufficient condition of realizability for this case is the following.

"Let finite real frequency transmission zeros be $0 \leq \sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_m < \infty$, then there should exist m+k-1 or more real frequency roots of $u_1(\lambda)$ that are not greater than σ_k. "

This is a condition similar to that stated by Fujisawa[7] on lumped ladder networks. It will be noted that a transmission zero at $\lambda = \infty$ is here not necessary. For its proof, $Y(\lambda)$ and $Y_{11}(\lambda)$ will be taken into consideration, where common factors of $Y_{11}(\lambda)$ are left uncancelled. Let

$$\left[\frac{Y(\lambda)}{\lambda}\right]_{\lambda=\infty} = C_{10},$$
(52)

then the shunt capacitance that can be taken out from $Y(\lambda)$ is

$$-\infty < C \leq C_{10}.$$
(53)

If $Y(\lambda)$ has no pole at ∞, C_{10} is of course taken to be 0. Suppose the allocation of zeros and poles of $Y_{11}(\lambda)$ is like Fig. (17). Then after taking out C, a zero
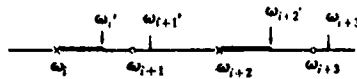
Fig. 17. Possible shifting interval of poles

say ω_{i+1}, can be moved to any point in the range $\omega_i < \omega \leq \omega_{i+1}'$. Here ω_{i+1}' is a position of a zero if C_{10} would have been taken out. Take out a cascade element, then the pole ω_i can be moved to any point in the range $\omega_{i-} \leq \omega \leq \omega_i'$. Here ω_i' is a position of a pole if C_{10} would have been taken out. Thus in the above procedure, any poles can be moved anywhere on the thick lines; and especially the highest pole can be moved up anywhere ranging to ∞. The larger the value of C to be taken out, the higher the points will move. Let the smallest among C_{10} and

$$
\begin{aligned}
C_{1\nu} &= \frac{Y(1) - j\sigma_\nu Y(j\sigma_\nu)}{1 + \sigma_\nu^2} \\
&= \frac{Y_{11}(1) - j\sigma_\nu Y_{11}(j\sigma_\nu)}{1 + \sigma_\nu^2},
\end{aligned}
$$
(54)

be $C_{1\mu}$. If any of $C_{1\nu}$ are negative, take the one of the greatest magnitude. Take out $C_{1\mu}$, then

$$Y_1(\lambda) = Y(\lambda) - \lambda C_{10}.$$ (55)

Take out a cascade element $Y_1(1)$ and obtain the remainder $Y_2(\lambda)$. Derive $Y_{11}(\lambda)$ corresponding to $Y_2(\lambda)$, then, from above considerations, the pole that was just below σ_μ in the original $Y_{11}(\lambda)$ will coincide with σ_μ, but other poles can not go up beyond transmission zeros. Therefore the relative allocation of poles and transmission zeros does not change through this procedure. Next, take out the pole at σ_μ and obtain $Y_3(\lambda)$. Derive $Y_{11}(\lambda)$ corresponding to $Y_3(\lambda)$. The pole σ_μ vanishes but the positions of other poles remains unchanged; that is, poles below σ_μ have the same ordinal relation, and as for transmission zeros above σ_μ, the number of poles of $Y_{11}(\lambda)$ will be decreased by 1 which do not exceed σ_μ. But, in the mean time, the transmission zero at σ_μ is also taken off. Renumber the transmission zeros from small to large, then there should be m+k-1 or more poles of $Y_{11}(\lambda)$ that do not exceed σ_μ. If $C_{1\mu} < 0$, then of course $Y_3(\infty) = 0$, so that a series inductance can be taken out. Besides, its value can be so chosen that it satisfy the condition of close coupling with $C_{1\mu}$, just as in the same way as in Bruner procedures. Thus, make

$$Y_4(\lambda) = - \cfrac{1}{\cfrac{1}{Y_3(\lambda)} - \lambda L}$$ (56)

then the corresponding pole of $Y_{11}(\lambda)$ will move a little upwards but cannot exceed the zero right above it. Take out a cascade element $Y_4(1)$ and make $Y_5(\lambda)$; the pole of corresponding $Y_{11}(\lambda)$ will again move upward but cannot exceed the original zero. Throughout the procedure from $Y_3(\lambda)$ to $Y_5(\lambda)$, poles of $Y_{11}(\lambda)$ move upward but never exceed poles right above. Therefore the number of poles, not higher than σ_k, can decrease, at most, by 1. That is, it can be stated, that there exist m+k-2 or more poles that do not exceed σ_k. On the other hand, the number of transmission zeros is now m-1, and if we rewrite the above number of poles as (m-1)+k-1, then it will be seen that the initial assumption is kept unaffected.

If, at the first procedure, C_{10} were the smallest then a transmission zero at ∞ will be taken off by taking out C_{10}. (If $C_{10} = 0$, it is only necessary to

take out a cascade element.) In this procedure none of the poles can exceed the corresponding σ_k. Thus the condition is never broken in any of the procedures, so that the synthesis can be continued, followed by a decrease of the degree of $Y_{11}(\lambda)$, until the whole procedure is completed in a finite number of steps.

In the above proof, no comment has been made on the number of cascade elements. It can be known as follows that

$$l \geq 2m+r-1 \tag{57}$$

suffices. Here it is assumed that $Y(\infty)$ is ∞ if ∞ is a transmission zero. If a finite pole is taken out at the first procedure, m decreases by 1, and l decreases by 2, and therefore the above relation remains unaffected. For the pole at $\lambda = \infty$, both l and r decrease by 1, and the above relation also remains unaffected. If, during the procedure, one gets to $r = 0$ and can no more take out finite transmission zeros, only a cascade element will be taken out, followed by a decrease of 1 only. But in this case there are no transmission zeros above the highest pole of $Y_{11}(\lambda)$ (if not, the transmission zero can be taken out), so that there must be at least 2m finite poles of $Y_{11}(\lambda)$, if the number of the remaining finite transmission zeros is m. Therefore the degree n of the K-matrix is at least 4m, and $n = 2m+l+r$, but in this case $r = 0$, and hence we have

$$l = n-2m, \cdot 2m \tag{57a}$$

It may be seen that the condition Eq. (57) still holds even if l is decreased by 1. Thus it has been proved that a network having the K-matrix that satisfies the above condition can always be constructed in a form shown in Fig. 18. Apply transformations described in the preceding chapter, it may be
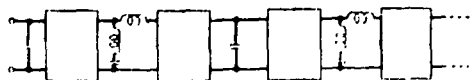

Fig. 18. Synthesis of a tree-and-branch network in a broader sense

transformed into one with only positive elements, as shown, for instance in
Fig. 19.
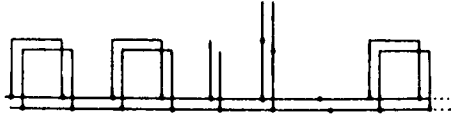
Fig. 19. A tree-and-branch network in a broader sense

In the above, realizability conditions on K-matrices were treated. Now
consider the case where only the input admittance $Y(\lambda)$ is given. It should
be noted that the number of cascade elements is not specified, and any number
of them can be used in the structure, which was not the case where K-matrices
were given. Of course in this case these cascade elements are all degenerate
except those specified in $Y(\lambda)$. Because of this freedom, the condition of
realizability becomes a very general one as stated below.

(Theorem) If, in an admittance

$$Y(\lambda) = \frac{v_2 + u_2}{v_1 + u_1} \tag{57b}$$

roots of $v_1 v_2 - u_1 u_2$ lie only on the imaginary axis, except the origin, and
on $\lambda = \pm 1$, then a tree-and-branch network, in a broader sense, exists that
has $Y(\lambda)$ as its input impedance.

To prove this theorem, it will be enough that among K-matrices of the
networks having $Y(\lambda)$ as input impedances (there are an infinite number of
such networks), some of them can be shown to satisfy the above conditions of re
realizability. For the prupose, the following two auxiliary theorems should
be proved.

(Auxiliary theorem 1) If n is taken large enough, it is possible to
put any number of roots of even and odd parts of $(1+\lambda)^n$ into any finite interval
$(0, j\sigma)$ on the imaginary axis.

The even part of $(1+\lambda)^n$ may be written

$$\{(1+\lambda)^n + (1-\lambda)^n\}/2 \tag{57c}$$

and with $\lambda = j\tan\theta$, this expression may also be written

$$\frac{e^{jn\theta} + e^{-jn\theta}}{2\cos^n\theta} = \frac{\cos n\theta}{\cos^n\theta},$$

(57d)

Its roots have the form $\theta = (2k+1)\pi/2n$. Thus the roots of the even part of $(1+\lambda)^n$ are

$$\pm j\tan\frac{(2k+1)\pi}{2n} \qquad \left(k = 0,\ 1,\ \cdots\cdots,\ \left[\frac{n}{2}\right]-1\right)$$

(58)

Similarly, those of the odd part are

$$0,\ \pm j\tan\frac{k\pi}{n} \qquad \left(k = 1,\ 2,\ \cdots\cdots,\ \left[\frac{n-1}{2}\right]\right)$$

(59)

The following auxiliary theorem can be directly deduced from this consideration.

(Auxiliary theorem 2) Let $p_1(\lambda)$ and $p_2(\lambda)$ be even polynomials and $q_1(\lambda)$ and $q_2(\lambda)$ be odd polynomials. If

$$\frac{p_1(\lambda)}{q_1(\lambda)},\qquad \frac{p_2(\lambda)}{q_2(\lambda)}$$

(59a)

are reactance functions, then the roots of

$$q(\lambda) = p_1(\lambda)q_2(\lambda) + p_2(\lambda)q_1(\lambda)$$

(60)

lie all on the imaginary axis, and the number of roots lying in the interval $(0, j\sigma)$ is equal to or one larger than the sum of numbers of roots of $q_1(\lambda)$ and $q_2(\lambda)$ lying in the same interval. Here a root at $\lambda = 0$ should be excluded and any roots of $q_1(\lambda)$ and $q_2(\lambda)$ that are coincident should be counted as 1.

Its proof can easily be seen from the fact that

$$\frac{p_1(\lambda)}{q_1(\lambda)} + \frac{p_2(\lambda)}{q_2(\lambda)} = \frac{p_1(\lambda)q_2(\lambda) + p_2(\lambda)q_1(\lambda)}{q_1(\lambda)q_2(\lambda)}.$$

(61)

is also a reactance function, and has poles of both q_1 and q_2, and there must be a zero between two poles.

Now, return to the theorem. Multiplying both the numerator and the denominator by $(1+\lambda)^s$, one obtains

$$Y(\lambda) = \frac{(v_2+u_2)(1+\lambda)^s}{(v_1+u_1)(1+\lambda)^s}$$

$$= \frac{(v_2p+u_2q)+(u_2p+v_2q)}{(v_1p+u_1q)+(u_1p+v_1q)} \tag{62}$$

where p and q stand for the even and odd parts of $(1+\lambda)^s$. Write this expression as

$$\frac{v_2'+u_2'}{v_1'+u_1'} \tag{62a}$$

then from the relation

$$v_1v_2-u_1u_2 = (1-\lambda^2)^s f(\lambda)^2 \tag{63}$$

one obtains the relation

$$v_1'v_2'-u_1'u_2' = (1-\lambda^2)^{s+s} f(\lambda)^2 \tag{64}$$

Here it will be assumed that the constant terms of v_1 and v_2, and consequently those of v_1' and v_2', are 1 (this assumption does not limit generality). Then the corresponding K-matrix can be written

$$\frac{1}{(\sqrt{1-\lambda^2})^{s+s} f(\lambda)} \quad \begin{matrix} v_1' & u_1' \\ u_2' & v_2' \end{matrix} \tag{65}$$

Now, let any root of $f(\lambda)$ be $j\sigma_k$, then the number of roots of $u_i(\lambda)$ that lie in the interval $(0, j\sigma_k)$ is not smaller than the sum of the numbers of roots of $u_1(\lambda)$ and $q(\lambda)$ in the same interval. However, since if s be taken great enough, the number of the roots of $q(\lambda)$ in the interval can be made as large as desired, so that also the number of roots of $u_i(\lambda)$ in the interval $(0, j\sigma_k)$ can be made as large as desired. On the other hand, multiplication by a factor $(1+\lambda)^s$ does not affect the number of transmission zeros at finite frequencies. Consequently, it is always possible to choose the value of s so

that the number of positive real roots of $u_i(\lambda)$ smaller than σ_k is equal to or greater than m+k-1. Therefore the network can be synthesized as a tree-and-branch network in a broader sense.

From this theorem, it has become always possible to realize a network, having transmission zeros only on the real frequency axis, except the origin and at $\lambda = \pm 1$, without using any negative elements, if a sufficient number of cascade elements are used. Moreover, a simple transmission zero can be located at the origin by the use of a shunt inductance. This status is rather more general than in the case of lumped ladder networks.

So far, only the cases were discussed where the transmission zeros lie all on the imaginary axis except $\lambda = \pm 1$. This is almost sufficient for practical purposes, as stated already, but it would be necessary to discuss more general cases from the standpoint of the theory of network synthesis. Even in this article, it has been shown that the synthesis is possible if the transmission zero lies on the real axis at a point of magnitude greater than 1. Those having transmission zeros on other points cannot be constructed if only the unit loops are permitted. A general normal network with larger loops will be necessary to be taken into consideration to treat those cases, but it is not the scope of this article. The author only refers to Ozaki's parallel tree-and-branch networks constitute simple examples therof.

5. APPROXIMATION OF CHARACTERISTICS

It is a very important and interesting problem how the transmission characteristics of a network can be approximated to a desired one, in the design of distributed filters or matching networks. In this chapter, the matters will be discussed one after the other.

First, the discussion will be confined to open-branch networks. Since $\lambda = 0$ comes in the pass band, they are appropriate for low-pass filters. and may also be used as band-pass filters if the periodicity of the characteristics is taken into consideration.

The operating transmission coefficient $S(\lambda)$ of a 4-terminal network is, by the network theory,

$$S(\lambda) = \frac{A+B+C+D}{2} \tag{66}$$

where the terminating admittances at both ends are taken to be 1. In open-branch networks, it may be written

$$S(\lambda) = \frac{v_1(\lambda)+v_2(\lambda)+u_1(\lambda)+u_2(\lambda)}{2(\sqrt{1-\lambda^2})^i f(\lambda)}$$

$$= \frac{g(\lambda)}{(\sqrt{1-\lambda^2})^i f(\lambda)} \tag{67}$$

where

$$g(\lambda) = \frac{v_1(\lambda)+v_2(\lambda)+u_1(\lambda)+u_2(\lambda)}{2} \tag{68}$$

is a Hurwitz polynomial with a constant term 1. Furthermore, put

$$h(\lambda) = \frac{v_1(\lambda)-v_2(\lambda)+u_1(\lambda)-u_2(\lambda)}{2} \tag{69}$$

then the echo transmission coefficients may be written

$$T(\lambda) = \frac{1+Y(\lambda)}{1-Y(\lambda)} = \frac{g(\lambda)}{h(\lambda)} \tag{70}$$

Here the condition Eq. (38) should be replaced by

$$g(\lambda)\,g(-\lambda)-h(\lambda)\,h(-\lambda) = (1-\lambda^2)^i f(\lambda)^2 \tag{71}$$

Conversely, if $S(\lambda)$ is given in the form Eq. (67), one can dertermine the polynomials coming into consideration from $S(\lambda)$. For the purpose, one should first determine $h(\lambda)$ (not always unique) from Eq. (71), and make $m(\lambda)$ and $n(\lambda)$ by

$$m(\lambda) = g(\lambda)-h(\lambda)$$
$$n(\lambda) = g(\lambda)+h(\lambda) \tag{72}$$

and then separate them into even and odd parts:

$$m(\lambda) = v_2(\lambda) + u_2(\lambda),$$
$$n(\lambda) = v_1(\lambda) + u_1(\lambda). \tag{73}$$

In most cases of approximation of characters, only the amplitude characteristics are specified. Therefore consider the expression

$$S\bar{S} = S(\lambda)\,S(-\lambda) = \frac{g(\lambda)\,g(-\lambda)}{(1-\lambda^2)^l\,f(\lambda)^2}$$

$$= 1 + \frac{h(\lambda)\,h(-\lambda)}{(1-\lambda^2)^l\,f(\lambda)^2} \tag{74}$$

which represents the square of the magnitude of operating transmission coefficient. It is desired to approximate this function to a desired characteristic. As for the value of l, it may be arbitrary, because if l is too small compared with the degree of $g(\lambda)$, one can multiply the numerator and the denominator by $(1-\lambda^2)^s$ before splitting $S(\lambda)$ from Eq. (74). If only the input admittance is required, the multiplication by this factor is not necessary. One should only obtain $g(\lambda)$ and $h(\lambda)$ from Eq. (74) itself and put

$$Y(\lambda) = \frac{m(\lambda)}{n(\lambda)} = \frac{g(\lambda) - h(\lambda)}{g(\lambda) + h(\lambda)}, \tag{75}$$

Although l has been said to be arbitrary, the number of cascade elements would become larger than l first assumed, which means the increase of degenerate elements that have not direct effect on the characteristcs. Therefore it may be better to assume the function with l as the number of necessary cascade elements.

Since $g(\lambda)$ is a Hurwitz polynomial, it is determined uniquely from Eq. (74), but $h(\lambda)$ is not always unique. Even when the numerator and the denominator is multiplied by $(1-\lambda^2)^s$, there may be various ways of sharing the factor to multiply into $h(\lambda)$. Take $(1+\lambda)$, then it means to insert a cascade section at the output terminals; take $(1-\lambda)$, then the same at the input terminals. By an appropriate choice, the network may be transformed into a desirable form.

Now the characteristics are specified only on the imaginary axis, it may be convenient to use $\omega = \lambda/j$ in place of λ itself. First, for the bar network,

the simplest, one has

$$SS = \frac{G(\omega)}{(1+\omega^2)^n},$$ (76)

where $G(\omega)$ is an even polynomial of degree 2n in ω, and $G(0)=1$. Now, by the transformation

$$\omega = \tan \frac{\theta}{2},$$ (77)

depict the interval $-\infty < \omega < \infty$ into an interval $-\pi < \theta < \pi$ (this means that the measure has been returned to one proportional to the frequency, that is $\theta = \pi f / f_0$). Then, since

$$\cos^2 \frac{\theta}{2} = \frac{1}{1+\omega}$$ (78)

Eq.(76) can be represented as a polynomial of degree n in $\cos^2 (\theta/2)$, and consequently also by a Fourier cosine series of n terms as follows:

$$SS = a_0 + a_1 \cos\theta + \cdots + a_n \cos n\theta.$$ (79)

Therefore the given characteristics should be approximated by a Fourier cosine series of n terms. To convert this series into an expression in ω, it will be convenient to use

$$\cos r\theta = \frac{(1-j\omega)^{2r} + (1+j\omega)^{2r}}{2(1+\omega^2)^r},$$ (80)

As alternate methods one may use the transformations

$$\chi = \frac{1}{1+\omega^2} \quad \text{or} \quad \frac{\omega^2}{1+\omega^2} \quad \text{or} \quad \frac{1-\omega^2}{1+\omega^2}$$ (80a)

and carry on approximations in terms of χ. Especially, if a low-pass filter is aimed, Wagner-character or Tchebycheff character will mostly be used. For the former, one should put

$$S\bar{S} = 1 + \frac{h^2\omega^{2n}}{(1+\omega^2)^n}$$ (81)

and for the latter

$$S\bar{S} = 1 + \frac{\delta}{2}\left[1 + \cos\left\{n\cos^{-1}\left(2\frac{\omega^2(1+\omega_1^2)}{\omega_1^2(1+\omega^2)} - 1\right)\right\}\right]$$

(82)

Here n is taken to be odd, and ω_1 is the limiting point of the pass band.

In case of simple open-branch networks, the form of $S\bar{S}$ is

$$S\bar{S} = \frac{G(\omega)}{(1+\omega^2)^l}$$

(83)

like Eq. (76), but here if $G(\omega)$ is an even polynomial of degree 2n, it is only necessary that n>1. Thus, if $A(\omega)^2$ is the characteristic to be approximated by $S\bar{S}$ (this may also be given graphically), one may determine $G(\omega)$ by approximating $A(\omega)^2 (1+\omega^2)^l$ by an even polynomial of degree 2n, or, instead, make $A(\omega)^2/(1+\omega^2)^{n-l}$ and approximate it by a Fourier series, as in the case of bar networks.

To have Wagner characteristics, one should put

$$S\bar{S} = 1 + \frac{h^2\omega^{2n}}{(1+\omega^2)^l}$$

(84)

To have Tchebycheff ones,

$$S\bar{S} = 1 + \frac{\delta}{2}\left[1 + \cos\left\{2(n-l)\cos^{-1}\frac{\omega}{\omega_1}\right.\right.$$
$$\left.\left. + l\cos^{-1}\left(2\frac{\omega^2(1+\omega_1^2)}{\omega_1^2(1+\omega^2)} - 1\right)\right\}\right]$$

(85)

Now, in general cases, one has a form

$$S\bar{S} = \frac{G(\omega)}{(1+\omega^2)^l F(\omega)}$$

(86)

If $l = 0$, the matter is just the same as in lumped ones; well established principles apply themselves and any filters of high class can be designed with Tchebycheff characteristics in the pass band as well as in the stop band. However, in this case, as stated already, all cascade elements are degenerate and have no effect on the characteristics, only increasing the delay time. In

other words, this way does not make full use of all possibilities of elements. However, it is still an unsolved problem to make the characteristic of a form of Eq. (86) into a Tchebycheff one in the pass band as well as in the stop band. Kasahara and Fujisawa[5], and Kuroda[8] made proposals on this point, but the writer has obtained a fairly satisfactory method by extending Bennett's theory. In the method by Kasahara and Fujisawa, first the form of

$$S\bar{S} = 1 + \frac{h(\lambda) h(-\lambda)}{f(\lambda)^2} \tag{87}$$

is determined from the lumped network theory. Then by choosing a correction term $M(\lambda) M(-\lambda)$, one obtains

$$S\bar{S} = 1 + \frac{M(\lambda) M(-\lambda) h(\lambda) h(-\lambda)}{f(\lambda)^2} \tag{88}$$

Here the correction term has a denominator $(1-\lambda^2)^\ell$, smaller than 1 in the pass band, and larger than 1 in the stop band; a Tchebycheff characteristic of a bar network may be used, for example. The characteristic thus obtained is not a perfect Tchebycheff, but is surely improved as compared with that of Eq. (87).

Next, Kuroda's method is an extension of that presented by the writer for bar networks and simple branch networks, and belongs to one of applying Darlington's image characteristic functions[18]. That is, let

$$S\bar{S} = 1 + \partial \cosh^2 \theta \tag{89}$$

and represent $\theta = \theta_1 + \theta_2 + \ldots + \theta_n$. Make θ_i such that $\cosh \theta_i$ are irrational reactance functions (Q-functions) with congruent real ranges and imaginary ranges. Some of them are chosen to have poles at $\lambda = \pm 1$. By this method a function will be obtained that has poles on any points, and has a Tchebycheff characteristic in the pass band. The method to be described below is also an extension of a method often used in lumped network theory[18][19], and is essentially identical with Kuroda's, but for calculation purposes, it will be more convenient. (Call it "χ-parameter" method).

By the transformation

$$x = \frac{\sqrt{\omega_1^2 + \lambda^2}}{\lambda} \qquad (90)$$

convert the pass band $(0, j\omega_1)$ into the imaginary axis $(-j\infty, 0)$ of χ, stop band $(j\omega_1, j\infty)$ on the real axis $(0, 1)$. Then

$$\varphi_1 = \left(\frac{x_1 + x}{x_1 - x}\right)^2 \qquad (91)$$

where

$$x_1 = \sqrt{\sigma_1^2 - \omega_1^2}/\sigma_1 \qquad (91a)$$

has a magnitude 1 in the pass band, is greater above $j\omega_1$, and becomes infinity at $j\omega_1$. Thus construct a function

$$\varphi = \prod_{\nu=1}^{m} \left(\frac{x_\nu + x}{x_\nu - x}\right)^2 \left(\frac{1 + x}{1 - x}\right)^r \left(\frac{x_0 + x}{x_0 - x}\right)^l \qquad (92)$$

where

$$x_\nu = \sqrt{\sigma_\nu^2 - \omega_1^2}/\sigma_\nu, \qquad x_0 = \sqrt{1 + \omega_1^2} \qquad (92a)$$

then it has also a magnitude 1 in the pass band, is infinity of multiplicity 2 at $\lambda = j\sigma_1$, $j\sigma_2, \ldots$, of multiplicity r at $\lambda = \infty$, and of multiplicity l at $\lambda = \pm 1$.

In the next place, make q from ϕ through

$$\varphi = \frac{1 + q}{1 - q} \qquad (93)$$

then q has the property of a reactance function in the pass band, takes a value between 0 and 1 above $j\omega_1$, and becomes 1 at each attenuation poles. Now make

$$\phi = \frac{1}{1 + q} = \frac{1 + \varphi^{-1}}{2} = \frac{1 + \bar{\varphi}}{2} \qquad (94)$$

then ψ is a characteristic function with a Tchebycheff behavior. That is, $\psi\bar\psi$ is positive real on the imaginary axis of λ, oscillates between 0 and 1 in the pass band, and have poles of multiplicity 2 at each σ_ν at ∞, and of multiplicity l at ± 1. With an appropriate constant h, make

$$S\bar{S} = 1 + h^2\psi\bar\psi \tag{95}$$

then a desired function of Tchebycheff behavior will be obtained. $\psi\bar\psi$ may also be obtained, with convenience, from

$$\psi\bar\psi = \frac{\{\Pi(x_\nu+x)^2(1+x)^r(x_0+x)^l \,\text{の偶数部}\}^2}{\Pi(x_\nu^2-x^2)^2(1-x^2)^r(x_0^2-x^2)^l}. \tag{96}$$

Thus a Tchebycheff characteristic function has been obtained with poles at arbitrary points. The next problem is to choose the position of these poles so that the characteristic is also Tchebycheff in the stop band. One method is to adopt a function known for lumped networks and add poles at ± 1. Of course this will not give a strict Tychebycheff character, but would be better than those by Kasahara and Fujisawa previously cited. It may be a method to draw graphs of $\log (\chi+\chi_\nu)/(\chi-\chi_\nu)$ for various χ_ν and construct $\log \phi$ by combining them graphically, just as could be done in image parameter design, combining various m-derived sections. But a perfect characteristic could not be expected by such an unsystematic way. In contrast to this, the method of successive approximation described below will give almost perfect Tchebycheff characteristics. This corresponds to an entension of the method given by B. J. Bennett[22] for lumped networks, and to obey the sequence, his method will be followed in short. Call a function $\psi\bar\psi$ for the sake of convenience a front-side function, which represents the square of the magnitude of a characteristic function having given pass and stop bands, and call a function with inverse pass and stop band a back-side function. If a front - side function is Tchebycheff in both bands, its reciprocal constitutes a back-side function, Tchebycheff in both bands. First make a front-side function which is Tchebycheff in the pass band with arbitrary poles in the stop

band. This is the 1st approximation. Obtain its zero points and make a back-side function which is Tchebycheff in the stop band with poles at the zero points of the 1st function. The reciprocal of this function gives the 2nd approximation. Again make a front-side function with poles at zeros of the back-side function. Repeat this procedure, and a characteristic will be attained which as near to the ideal as desired. The convergence of this procedure is fairly fast, and it is satisfactorily practicable. Thus a function can be obtained which is Tchebycheff in both bands, without using elliptic functions.*

One may try this method to distributed networks but he will find that it does not go well by itself. The reason is as follows. Since attenuation poles exist at $\lambda = \pm 1$, the back-side function must be 0 at these points, but it is not possible to make a characteristic function with zeros at arbitrary points. But if the idea of potential theory is made use of, a function can be made, which has a good approximation. The method will be explained from the beginning.

Let the pass band be $0 \sim \omega_1$, stop band $\omega_2 \sim \infty$, and poles at $\lambda = \pm 1$ with a multiplicity l. Assume transmission zeros j $\sigma_1, \ldots$, in an arbitrary way, and make

$$\varphi = \prod_{\nu=1}^{m} \left(\frac{x_\nu + x}{x_\nu - x} \right)^2 \left(\frac{x_0 + x}{x_0 - x} \right)^l \tag{97}$$

in a similar manner as before. Make $\psi \overline{\psi}$ from this ψ, then the first approximate front-side function will be obtained. The only needed are the roots of $\psi = 0$ and therefore (with μ an integer) from $\phi = -1$, that is, from

$$2 \sum_{\nu=0}^{m} \tan^{-1} \frac{x}{j x_\nu} + l \tan^{-1} \frac{x}{j x_0} = (2\mu+1) \frac{\pi}{2} \tag{98}$$

the roots $\chi_{01}, \chi_{02}, \cdots \chi_{0n}$ will be obtained (to facilitate the calculation, one should make use of a table of trigonometric functions). The corresponding values of λ will give the zeros of the front-side function. To make a back-side function that has poles at these zeros, first define y by

$$y = \frac{\sqrt{\lambda^2 + \omega_2^2}}{\omega_1} = \sqrt{\frac{(1-k^2)-x^2}{1-x^2}}, \tag{99}$$

*(Translator's note: This method of successive approximation was further extended by K. Hatori.)

where

$$k = \omega_1 / \omega_2 \tag{99a}$$

and let the values of y be y_{01}, y_{02}, $\cdots$ that correspond to χ_{01}, χ_{02}, $\cdots$. Make

$$\varphi' = \prod_{\nu=1}^{n} \left(\frac{y_{0\nu}+y}{y_{0\nu}-y} \right)^2 \left(\frac{y_0-y}{y_0+y} \right)' \tag{100}$$

where $y_0 = \sqrt{1 + \omega_2^2} / \omega_2$. Thus ϕ' has zeros at $y = y_0$, i.e. at $\lambda = \pm 1$, but upon making $\psi'\overline{\psi}'$, $y=y_0$ becomes a pole of $\psi'\overline{\psi}'$, and one cannot have a back-side function. Obtain zeros y_1, y_2, $\cdots y_m$ of $\psi'\overline{\psi}'$ on the imaginary axis and construct a function

$$\psi''\psi'' = \frac{\prod_{\nu=1}^{m} (y_\nu^2 - y^2)^2 (y_\nu^2 - y^2)'}{\prod_{\nu=1}^{n} (y_{0\nu}^2 - y^2)^2} \tag{101}$$

then this function has a zero at $y = y_0$, poles at $y_{0\nu}$, and is almost Tchebycheff on the imaginary axis of y (stop band in λ). The reason can be explained with an application of the potential theory[23], as seen in the following. Consider $\log \phi'$ to be a complex potential on the λ-plane. As seen from Eq. (99), two Riemann surfaces can be considered to be connected at the stop band on the imaginary axis. Place a line conductor on this portion, and put a shield between the two surfaces. If we consider only one side of the surface, there will be a continuous distribution of electric charge on the conductor. The distribution of charge is determined by the flow-function

$$\text{imag. } \log \varphi' = \arg \varphi' \tag{101a}$$

The charge contained between any two points in the stop band is equal to one π - th of the difference of arg ϕ' on the right side of the two points. Quantize this charge. For this purpose divide the conductor into portions each with 2 units of charge, and place these charges at the centers of each portions. Through this procedure, the potential, where the conductor was originally located, is almost Tchebycheff. The locations of quantized charges are such that arg ϕ' is odd multiples of π, i.e.,

$$2 \sum_{\nu=1}^{\eta} \tan^{-1} \frac{y}{jy_{0\nu}} - l \tan^{-1} \frac{y}{jy_0} - (2\mu+1)\frac{\pi}{2} \qquad (102)$$

This gives nothing but the $y_1 \ldots, y_m$ formerly stated. Thus it can be seen that Eq. (101) is approximately Tchebycheff. In that way the 2nd back-side function is obtained. To proceed further, one should make a front-side function with poles at y_ν. A numerical example will be shown:

$$\omega_1 = 1, \quad \omega_2 = \sqrt{2}, \quad m = 3, \quad l = 5. \qquad (102a)$$

Start with all poles at $\lambda = \infty$. Table 1 is the result. y's are converted into χ's for back-side functions.

Table 1. An example of succesive approximations

		0	2	4	6
Back-Side	x_1	1	0.7186	0.71823	0.71821
	x_2	1	0.8080	0.80545	0.80543
	x_3	1	0.9583	0.95606	0.95603
		1	3	5	7
Front-Side	x_{01}	j 0.1659	j 0.1456	j 0.14540	j 0.14540
	x_{02}	j 0.5294	j 0.4682	j 0.46758	j 0.46757
	x_{03}	j 1.0121	j 0.9069	j 0.90582	j 0.90581
	x_{04}	j 1.8339	j 1.6640	j 1.66349	j 1.66346
	x_{05}	j 4.0378	j 3.7054	j 3.70217	j 3.70211
	x_{06}	$j \infty$	$j \infty$	$j \infty$	$j \infty$

One will see that approximation is fairly good at the third. Transform the front-side function obtained from the 7th approximation into that of λ, one has

$$\psi\bar{\psi} = K \frac{-\lambda^2 (0.97950+\lambda^2)^2 (0.82060+\lambda^2)^2 (0.54950+\lambda^2)^2 (0.26546+\lambda^2)^2}{(2.06545+\lambda^2)^2 (2.84677+\lambda^2)^2 (11.6287+\lambda^2)^2}$$
$$\times \frac{(0.068001+\lambda^2)^2}{(1-\lambda^2)^5} \qquad (102b)$$

The deviation of minimum attenuation in the stop band is within 0.1 db, and can be considered to be perfectly Tchebycheff.

To add words to the χ-parameter method, although needless to say, those corresponding to Eqs. (82) and (85) can be obtained also as special cases.

In the last place, citations will be made on matching networks. Heretofore, both terminating admittance were taken to be 1, but here they are assumed not to be equal. Let these be G_1 and G_2, and also

$$G = \sqrt{G_1 G_2}, \qquad k = \sqrt{\frac{G_1}{G_2}} \qquad (103)$$

The frequency band of matching required is assumed to be (f_1, f_2) as shown in Fig. 20, with f_0 at the center. Obtain λ from Eq. (1). Since the
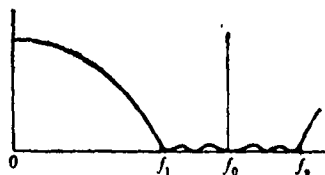

Fig. 20. The characteristic of a matching network

characteristic is periodical, it is only necessary to have a good match in the interval (f_1, f_0). In terms of λ, it is to have a match in $(j\omega_1, j\infty)$ where

$$\omega_1 = \tan \frac{\pi f_1}{2 f_0} \qquad (103a)$$

This is a kind of a high-pass filter, but since there are no requirements on the attenuation in mismatch ranges, so that no finite frequency transmission zeros are necessary, and a network of a simple configuration will be satisfactory. Shunt capacitances cannot be used because $\lambda = \infty$ is in the pass band. It follows that a bar network or a simple short-branch network will be the representative ones.

Now, $S(\lambda)$ for different terminating admittance should be written

$$S(\lambda) = \frac{kA + GB + G^{-1}C + k^{-1}D}{2} \qquad (104)$$

and therefore for a bar network

$$S(\lambda) = \frac{g(\lambda)}{(\sqrt{1 - \lambda^2})^n} \qquad (105)$$

where

$$g(\lambda) = kv_1 + Gu_1 + G^{-1}u_2 + k^{-1}v_2 \tag{106}$$

and in this case one has

$$S(0) = g(0) = \frac{k + k^{-1}}{2} \geq 1 \tag{107}$$

Conversely, to obtain the polynomials when $g(\lambda)$ is known, one has to proceed as stated before. First obtain $h(\lambda)$ from

$$g(\lambda)\,g(-\lambda) - h(\lambda)\,h(-\lambda) = (1 - \lambda^2)^n \tag{108}$$

where it is so chosen that

$$h(0) = \frac{k - k^{-1}}{2} \tag{109}$$

From this obtain $m(\lambda)$ and $n(\lambda)$ by Eq. (72), and separate them into even and odd parts respectively:

$$m(\lambda) = kv_2(\lambda) + G^{-1}u_2(\lambda),$$
$$n(\lambda) = k^{-1}v_1(\lambda) + G\,u_1(\lambda). \tag{110}$$

As for approximation of the characteristics, λ should be replaced by $1/\lambda$ in the foregoing representations, because $\lambda = \infty$ is at the center of the pass band. But the parameters should be so determined that the condition Eq. (107) is satisfied.

For a Wagner characteristic, one shall put

$$S\bar{S} = 1 + \frac{(k - k^{-1})^2}{4(1 - \lambda^2)^n} \tag{111}$$

Let n= 1, for instance, then one has

$$Y_{11} = \frac{G}{\lambda} \tag{111a}$$

which means a unit element G, and is the same obtained elsewhere hitherto, because $G = \sqrt{G_1 G_2}$.

Let n = 2, then one has

$$G_n = \sqrt{k}\, G. \qquad G_h = \frac{G}{\sqrt{k}} \tag{111b}$$

and since there is a relation

$$\log \frac{G_1}{G_n} : \log \frac{G_n}{G_h} : \log \frac{G_h}{G_2} \quad 1 : 2 : 1 \tag{111c}$$

the result is the same as obtained so far. It was anticipated that even when n is equal to or larger than 3, the sucessive ratios, taken in a similar way, would be in the relation of binomial coefficients[10]. But calculations do not follow the anticipations. The anticipation seems to hold if k is nearly equal to 1.

To have a Tchebycheff character,

$$S\bar{S} = 1 + \frac{\partial}{2}\left[1 + \cos\left\{ n \cos^{-1}\left(2\, \frac{1+\omega_1^2}{1-\lambda^2} - 1 \right) \right\} \right] \tag{112}$$

will do. In order that Eq. (107) be satisfied, δ should be determined from

$$\frac{(k-k^{-1})^2}{2} = \partial[1 + \cos\{n \cos^{-1}(1 + 2\omega_1^2)\}] \tag{113}$$

Or, if one would follow the χ-parameter method, he has to put

$$x = \frac{\sqrt{\omega_1^2 + \lambda^2}}{\omega_1}, \qquad x_1 = \frac{\sqrt{\omega_1^2 + 1}}{\omega_1} \tag{114}$$

and make

$$S\bar{S} = 1 + \delta\, \frac{\{(x_1+x)^n \ \text{の偶数部}\}^2}{(x_1^2 - x^2)^n} \tag{115}$$

Since it is the magnitude of the reflected wave that is of major problem in a matching network, the echo transmission coefficient $T(\lambda)$ will be of more direct significance than the operating transmission coefficient. The minimum echo attenuation will be specified to be a certain db in the matching band. Needless to say, there is a relation between $T(\lambda)$ and $S(\lambda)$ such that

$$(S\bar{S})^{-1}+(T\bar{T})^{-1}=1 \qquad . \qquad (116)$$

and hence Eq. (115) can be rewritten as

$$T\bar{T}= 1+\frac{1}{\partial}\frac{(x_1{}^2-x_2{}^2)^n}{\{(x_1+x_2)^n \ \text{の偶数部}\}^2} \qquad (117)$$

Fig. 21, shows the relation among k^2, f_1 and n for the minimum echo
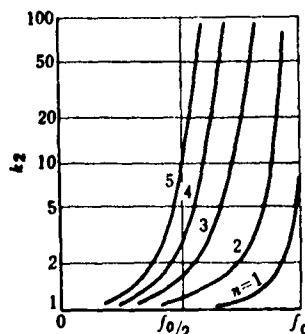

Fig. 21. Design chart for bar matching networks (minimum echo attenuation
        30 db)

attenuation of 30 db.

Next, for a network of simple short-branch type, $S(\lambda)$ takes the form

$$(118)$$

$$S(\lambda) = \frac{g(\lambda)}{\lambda(\sqrt{1-\lambda^2})^n}$$

and to find the polynomals, one should make use of the relations

$$(119)$$

$$g(\lambda) g(-\lambda)-h(\lambda) h(-\lambda) = -\lambda^2 (1-\lambda^2)^n$$

$$m(\lambda) = G^{-1} w_2(\lambda)+k\lambda v_2(\lambda),$$
$$n(\lambda) = G\lambda u_1(\lambda)+k^{-1}\lambda v_1(\lambda), \qquad (120)$$

Here it should be noted that the constant terms of $V_1(\lambda)$ and $V_2(\lambda)$ are equal to or
or greater than 1. But since there are no means to ascertain whether this
condition is satisfied or not, when determining $g(\lambda)$, one should see it in
invidual cases.

Now the approximation of the characteristics should be considered.
Since, in general,

$$S\bar{S} = \frac{g(\lambda) g(-\lambda)}{-\lambda^2(1-\lambda^2)^n} = 1+ \frac{h(\lambda) h(-\lambda)}{-\lambda^2(1-\lambda^2)^n} \qquad (121)$$

for a Wagner character one should take

$$SS = 1 - \frac{h^2}{\lambda^2(1-\lambda^2)^n}$$ (122)

For a Tchebycheff character,

$$S\bar{S} = 1 + \frac{\delta}{2}\left[1 + \cos\left\{2\cos^{-1}\frac{j\omega_1}{\lambda} + n\cos^{-1}\left(2\frac{1+\omega_1^2}{1-\lambda^2}-1\right)\right\}\right]$$ (123)

or

$$S\bar{S} = 1 + \delta\frac{\{(1+x)(x_1+x)^n \; \text{の偶数部}\}^2}{(1-x^2)(x_1^2-x^2)^n}$$ (124)

In these cases, h, δ, etc., should be so determined that the constant terms of v_1 and v_2 are equal to or greater than 1.

6. SYNTHESIS THEORY OF DISTRIBUTED TWO-TERMINAL NETWORKS

In this chapter the focus of interest will be shifted on the synthesis of 2-terminal networks. This problem has a certain concern with synthesis of 2-terminal networks without mutual inductance in case of lumped networks.

In case of lumped networks, theories are known as those given by Bott and Duffin[11], by Kiyasu[12] and by Miyata[13] which correspond to the extension of Bott and Duffin's. But all of these could not be applied to distributed networks in original unmodified forms. Only the idea used by Miyata may be applied, and the method below follows his idea.

The principle lies in the dissociation of a network into a number of partial networks with reference to the even part of the admittance. In case of a lumped networks, ladders of L and C are used as partial networks, whereas in case of distributed ones, instead, tree-and-branch networks in a broader sense are used, like those described in previous chapters.

If the admittance has poles on the imaginary axis, then they can be easily taken off as shunt reactances. Therefore the given admittance may be taken to have no poles on the imaginary axis from the beginning. An

admittance with no poles on the imaginary axis can be uniquely specified by its even part, as the network theory says. Divide this even part into a sum of a number of even functions, each of which satisfies the condition of the even part of an admittance (that is, non-negative on the imaginary axis). Then an admittance will be determined, for each of these, that has the specified even part, with no poles on the imaginary axis. The sum (i.e., the parallel connection) of these admittances will be equal to the original, from the uniqueness above stated.

The input admittance of a 4-terminal network with an admittance G at the end is

$$Y(\lambda) = Y_{11}(\lambda) - \frac{Y_{12}(\lambda)^2}{Y_{22}(\lambda)+G},$$

(125)

and therefore its even part becomes

$$G(\lambda) = \frac{1}{2}\{Y(\lambda)+Y(-\lambda)\} = \frac{-GY_{12}(\lambda)^2}{G^2-Y_{22}(\lambda)^2},$$

(126)

That is, $G(\lambda)$ is determined only by $Y_{12}(\lambda)$ and $Y_{22}(\lambda)$. However, since $Y(\lambda)$ has no poles on the imaginary axis, the residues of Y_{12} and Y_{22} are compact, and $Y_{11}(\lambda)$ is determined necessarily from these. Now, suppose the 4-terminal network were of a tree-and-branch type. Then it can be written that

$$Y_{22}(\lambda) = \frac{v_1(\lambda)}{u_1(\lambda)}, \qquad Y_{12}(\lambda) = \frac{(\sqrt{1-\lambda^2})' f(\lambda)}{u_1(\lambda)},$$

(127)

so that the polynomial defined by

$$n(\lambda) = v_1(\lambda)+Gu_1(\lambda),$$

(128)

is a Hurwitz polynomial with a constant term 1, and hence

$$G(\lambda) = \frac{G \cdot (1-\lambda^2)' f(\lambda)^2}{n(\lambda)\, n(-\lambda)},$$

(129)

Conversely, if $G(\lambda)$ has zeros only on the imaginary axis excluding origin and on ± 1, as shown in Eq. (129), then it is possible to make a tree-and-branch network (in a broader sense) having such an even part. Here, if ℓ, the number of the cascade element is insufficient, $Y_{22}(\lambda)$ and $Y_{12}(\lambda)$ may be determined after multiplying a factor $(1-\lambda^2)^s$ into the numerator and the denominator of Eq. (129).

Now, make the even part from the given admittance, it may in general be written in the following form

$$G(\lambda) = \frac{\sum\limits_{i=0}^{n} a_i \lambda^{2i}}{n(\lambda)\, n(-\lambda)}. \tag{130}$$

where $n(\lambda)$ is a Hurwitz polynomial with a constant term 1 and its degree is not smaller than n. Reexpand the numerator in terms of $(1 - \lambda^2)$, one has

$$G(\lambda) = \sum\limits_{i=0}^{n} \frac{b_i\, (1-\lambda^2)^i}{n(\lambda)\, n(-\lambda)}. \tag{131}$$

It may be seen that this can be realized by a parallel connection of simple open-branch networks, if all the coefficients b_ℓ is positive.

Even if some of b_ℓ are negative, they may be made positive, in most cases, by Miyata's method of "degree ascending". The method is as follows: Let $\phi(\lambda)$ be a Hurwitz polynomial. Multiply $\phi(\lambda)\,\phi(-\lambda)$ into the numerator and the denominator of $G(\lambda)$. Re-arrange the numerator in the form Eq. (131). Choose $\phi(\lambda)$ so that all the coefficients come out positive. In practical execution, it will be convenient to use $\chi = 1 - \lambda^2$, and represent the numerator in terms of X, and express also the polynomial $\phi(\lambda)\,\phi(-\lambda)$ as one in χ with no roots in $\chi \geq 1$. Unfortunately, however, this method is not always applicable. The necessary and sufficient condition of possibility of turning the coefficients of polynomials into positive is, in general, that "the coefficient of the term of the highest degree is positive, and the polynomial has no positive real roots"[14]. The proof is simple, but not referred here. The above condition, is stated in terms of λ, says "$G(\lambda)$ has no zeros on the imaginary axis or between ± 1 on the real axis." Roots on the imaginary axis may be expressed by

admittance with no poles on the imaginary axis can be uniquely specified by
its even part, as the network theory says. Divide this even part into a
sum of a number of even functions, each of which satisfies the condition of the
even part of an admittance (that is, non-negative on the imaginary axis).
Then an admittance will be determined, for each of these, that has the
specified even part, with no poles on the imaginary axis. The sum (i.e.,
the parallel connection) of these admittances will be equal to the original,
from the uniqueness above stated.

The input admittance of a 4-terminal network with an admittance G at
the end is

$$Y(\lambda) = Y_{11}(\lambda) - \frac{Y_{12}(\lambda)^2}{Y_{22}(\lambda)+G},$$

(125)

and therefore its even part becomes

$$G(\lambda) = \frac{1}{2}\{Y(\lambda)+Y(-\lambda)\} = \frac{-GY_{12}(\lambda)^2}{G^2-Y_{22}(\lambda)^2},$$

(126)

That is, $G(\lambda)$ is determined only by $Y_{12}(\lambda)$ and $Y_{22}(\lambda)$. However, since
$Y(\lambda)$ has no poles on the imaginary axis, the residues of Y_{12} and Y_{22} are
compact, and $Y_{11}(\lambda)$ is determined necessarily from these. Now, suppose
the 4-terminal network were of a tree-and-branch type. Then it can be
written that

$$Y_{22}(\lambda) = \frac{v_1(\lambda)}{u_1(\lambda)}, \qquad Y_{12}(\lambda) = \frac{(\sqrt{1-\lambda^2})' f(\lambda)}{u_1(\lambda)},$$

(127)

so that the polynomial defined by

$$n(\lambda) = v_1(\lambda)+Gu_1(\lambda),$$

(128)

is a Hurwitz polynomial with a constant term 1, and hence

$$G(\lambda) = \frac{G \cdot (1-\lambda^2)' f(\lambda)^2}{n(\lambda) n(-\lambda)}.$$

(129)

The network obtained is shown in Fig. 22 and its characteristic in

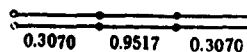0.3070   0.9517   0.3070

Fig. 22. A bar filter

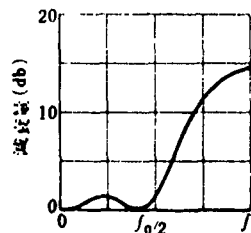Fig. 23. The abscissa is taken in the frequency f itself, here and hereafter.

2) A simple open-branch filter

$\omega_1 = 1$, $\delta = 0.2$ $(b_\rho \approx 0.8$ db$)$

Number of shunt elements $r = 2$

Number of cascade elements $l = 3$

Fig. 23. ordinate: Attenuation (db)

$$S\bar{S} = (1 - 10.79411\lambda^2 - 75.79798\lambda^4 - 270.43313\lambda^6$$
$$- 355.22522\lambda^8 - 158.39596\lambda^{10}) \div (1 - \lambda^2)^3,$$

(133b)

$$Y_{22}(\lambda) = \frac{1 + 12.00255\lambda^2 + 13.01421\lambda^4}{8.69088\lambda + 34.95359\lambda^3 + 25.17109\lambda^5}.$$

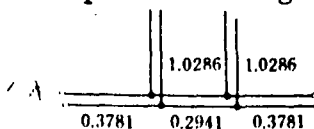The network is shown in Fig. 24. Adopt another sign of $h(\lambda)$ (i.e., the inverse

1.0286   1.0286

0.3781   0.2941   0.3781

Fig. 24. A simple open-branch filter (1)

network), then

$$Y_{22}(\lambda) \cdot \frac{1 + 12.00255\lambda^2 + 13.01421\lambda^4}{3.10729\lambda + 6.72874\lambda^3} \cdot$$

(133c)

and one obtains a network in Fig. 25. The characteristics of these are shown

1.9341   1.9341

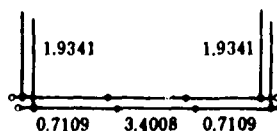0.7109   3.4008   0.7109

Fig. 25. A simple open branch filter (2)

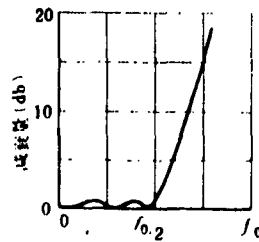in Fig. 26.

Fig. 26. ordinate: Attenuation (db)

3) A tree-and-branch type filter

Cutoff $\omega_c = 1$, $b_\rho = 0.5$ db

Minimum attenuation in stop band $b_s = 50$ db

Bandwidth factor $k_o > 0.75$

These requirements are satisfy by taking $n = 6$ and $k = 0.8$.

$$SS \quad 1 + h^{\prime\prime}_{1} \left[\frac{\lambda^2(\lambda^2 + \Omega_2^2)}{(1 + \Omega_2^2 \lambda^2)} \frac{(\lambda^2 + \Omega_4^2)}{(1 + \Omega_4^2 \lambda^2)} \right]^2,$$

$$\Omega_2 \quad 0.659352, \quad \Omega_4 \quad 0.85547$$

$$h^{\prime\prime} \quad 100 \tag{133d}$$

It should be noted that, upon taking out elements from the input admittance, the effective figures decreases on and on, so that it is needed to take sufficient decimal places at the beginning. This does not mean that Ω_2, Ω_4, etc., should be determined precisely, but that, if they were once determined, the error of calculation followed afterwards should be as small as possible. In the above example, five zeros were supplemented after each effective figures of Ω_2 and Ω_4, as if their effective figures were of 11 digits.

$$e(\lambda) \quad \lambda^6 + 1.27783265572\lambda^5 + 1.98244960014\lambda^4$$

$$+ 1.56199694949\lambda^3 + 1.02914466687\lambda^2$$

$$+ 0.42721014412\lambda + 0.10000000000, \tag{133e}$$

$$h(\lambda) \quad -\lambda^6 - 1.16602145515\lambda^4 - 0.31791879987\lambda^2.$$

An example of the network obtained is shown in Fig. 27. It is also
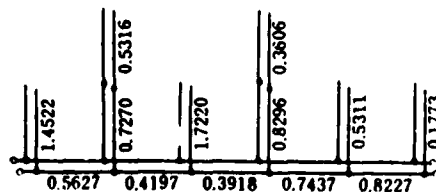

Fig. 27. A tree-and-branch filter

realizable in other order of transmission zeros. The characteristic is shown in Fig. 28. Since, in this case, it contains 5 cascade elements a variety of
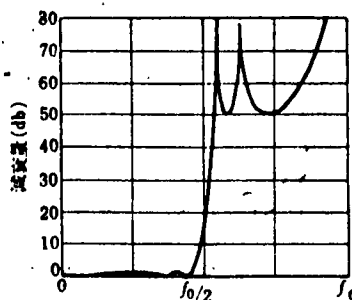
Fig. 28. ordinate: Attenuation (db)

networks will be obtained by multiplying $g(\lambda)$ by $(1+\lambda)^s$ ($s \leq 5$) and $h(\lambda)$ by $(1-\lambda)^s$.

4) A tree-and-branch filter in a broader sense.

$\omega_1 = 1$, $b_\rho = 0.4$ db, $b_s = 33$ db

$$S\bar{S} = 1 + \frac{2000\lambda^4 (\lambda^2+0.87)^2}{(\lambda^2+2.25)^2 (\lambda^2+16)^2},$$

$$g(\lambda) = 1+2.1328187\lambda+2.7813922\lambda^2$$

$$+2.0491042\lambda^3+1.2425705\lambda^4,$$

$$h(\lambda) = 1.0807662\lambda^2+1.2422600\lambda^4.$$

(133f)

Its configuration and characteristic are shown in Figs. 29 and 30.

Fig. 29. A tree-and-branch filter in a broader sense
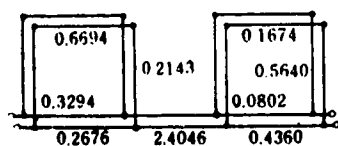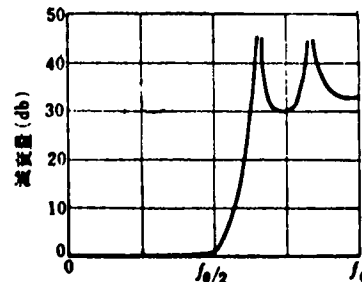

Fig. 30. ordinate: Attenuation (db)

5) An example using the χ-parameter

First, adopt a characteristic given in lumped networks.
$\omega_c = 1$, $k = 0.5$ (consequently $\omega_1 = 1/\sqrt{2}$)

$$S\bar{S} = 1 - 10.97633 \frac{\lambda^2(\lambda^2+0.388108)^2}{(0.388108\lambda^2+1)^2},$$

(133g)

The network configuration is as shown in Fig. 31, but it can be changed
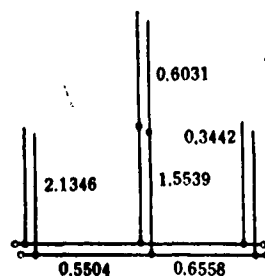
Fig. 31.

in a symmetrical one, Fig. 32, by multiplying g(λ) by 1+λ and h(λ) by 1-λ.
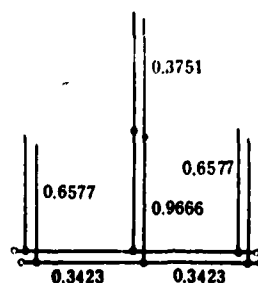
Fig. 32.

Their characteristic is shown in Fig. 33.

Fig. 33. ordinate: Attenuation (db)

Here, let us make a network with the same transmission zero, where the cascade elements have contributions to the characteristic.

$$x = \frac{\sqrt{\lambda^2 - 0.5}}{\lambda},$$

$$\varphi = \frac{(0.897745 + x)^2 (1+x)(\sqrt{1.5} + x)^2}{(0.897745 - x)^2 (1-x)(\sqrt{1.5} - x)^2}.$$

(133h)

from which one has

$$S\bar{S} = 1 - \frac{1016.77\lambda^2(\lambda^2+0.163616)^2(\lambda^2+0.449596)^2}{(0.388108\lambda^2+1)^2(1-\lambda^2)^3} \qquad (133i)$$

The network configuration is shown in Fig. 34, and its characteristic in
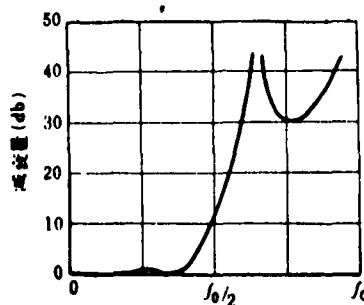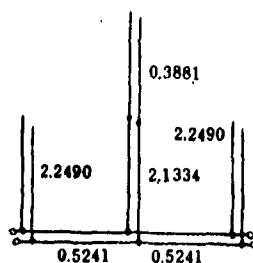

Fig. 34. Design by the χ-parameter method

Fig. 35. One will see how the characteristic has been improved.
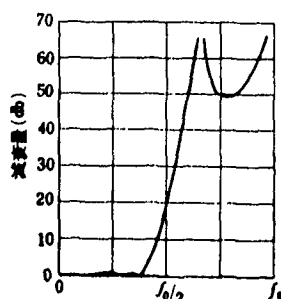
Fig. 35. ordinate: Attenuation (db)

6) A bar-type matching network

Let $G_1 = 1$, $G_2 = 10$, $\omega_1 = 1$ and Tchebycheff behavior of $n = 3$. Then one has $\delta = 0.0405$ (minimum echo attenuation $b_t = 14$ db), and

$$S\bar{S} = \frac{3.025 - 0.57\lambda^2 + 3.729\lambda^4 - \lambda^6}{(1-\lambda^2)^3} \qquad (133j)$$

The values of the three elements are 1.6347, 3.1623 and 6.1173 from left to right.

7) A simple short-branch matching network

Given requirements are $G_1 = 1$, $G_2 = 10$, $\omega_1 = 3$ $(f_1 = 2f_0/3)$ and $\delta = 0.01$ $(b_t = 20 \text{ db})$.

n = 2 will be sufficient for the requirements, so that one has

$$SS \cdot 1 + 0.005 \frac{\{(4\sqrt{2} + \sqrt{6})\lambda'' + 12\sqrt{2} + 7\sqrt{6}\}^2}{-\lambda''(1-\lambda^2)^2} \qquad (133k)$$

The configuration for this case is that shown in Fig. 36, but upon
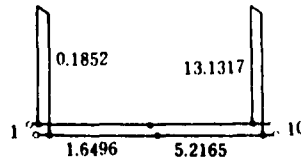
Fig. 36. A simple short-branch matching network (1)

changing the position of taking out a shunt inductance, one will obtain a network Fig. 37, in which the discrepancy of the element values is smaller than the former to be more profitable in construction.
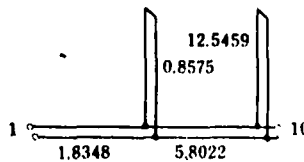
Fig. 37. A simple short-branch matching network (2)

8. CONCLUSIONS

It has been described about the synthesis of distributed networks with prescribed characteristics and also about approximation of the characteristics. In specifying the characteristics, only operating transmission coefficients were used, but it is also possible to specify them in terms of voltage transmission coefficients, of current transmission coefficients or of image parameters. In either case, one should obtain the Y-matrix or the input impedance, and follow the synthesis in like manners. In Chapter 6, it has been described to determine a network for a specified even part of its admittance. This is, in fact, of equal in effect to specify the voltage transmission coefficient.

In Chapter 5, the description was chiefly on low-pass filters, and only
the matching networks were discussed as high-pass filters, but it is also
possible to design high-pass filters with transmission zeros at finite
frequencies. Here the transmission zero at $\lambda = 0$ is necessarily simple, and
the network may be constructed with shunt inductances, unit loops and cascade
elements. The so-called high-pass filters become band-pass if used in the
second or higher modes. One should choose a type to fit his aim.

In Chapter 7, there are shown only those that have wide bands. This is the
feature of the procedure. It was only possible to obtain narrow band ones
by simple methods hitherto known. On the contrary, a narrow band networks,
designed by the new method, would include elements of very large or very
small characteristic admittances leading to difficulty of construction. It
would be desirable to construct a network with all equal characteristic
admittance (with different lengths), but it would need a thoroughly different
system of theorem from that of this article.

In the last place, it should be taken into consideration that the junctions
of elements do not act as ideal ones, but the problem should be solved in
parallel with experiments.

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Radiation, Inc.
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Ottawa, Ontario, Canada
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Microwave Section

Space Technology Labs., Inc.
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Los Angeles 45, Calif.
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Information Services.

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The Rand Corp.
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Chicago 16, Ill.
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Lawrence, Kansas
Attn: Dr. H. Unz

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Space Sciences Lab
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Berkeley 4, Calif
Attn: Dr. Samuel Silver,
     Prof. of Eng. Science
     and Dir., Space Sciences Lab.

4

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San Diego 12, Calif.
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Army Rocket and Guided Missile
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Redstone Arsenal, Ala.
Attn: Tech. Library, ORDXR-OTL

National Aeronautical Space
Agency, Langley Aeronautical
Research Lab.
Langley, Virginia
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National Bureau of Standards
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U. S. Army Signal Eng. Labs.
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Eng. and Technical Div.
Wash 25, D.C.
Attn: SIGNET-5

National Bureau of Standards
U.S. Dept. of Commerce
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